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ESTIMATION OF RESIDENTIAL WATER DEMAND FUNCTIONS FOR GREATER VICTORIA: AN EXPANSION ON THE THEORY AND TESTING OF THE ECONOMETRIC MODEL

Prepared by:
Sergio Gai

February, 1989

Inland Waters
Pacific and Yukon Region
Vancouver, B.C.

ENVIRONMENT CANADA

CONSERVATION AND PROTECTION

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ABSTRACT

This report presents the results of an extension of an earlier project (Gai 1988) to estimate residential water demand functions for the municipalities of Victoria, Esquimalt, Oak Bay and Saanich, collectively referred to as Greater Victoria, in British Columbia. As a result of the findings of the earlier study, this study expanded on the theoretical and econometric specifications of the model. A linear model and a comprehensive model derived from economic theory were tested against the log-log model previously used. Static and dynamic versions of each model were tested. Dynamic models of the flow-adjustment and stock-adjustment type were evaluated. Explanatory variables not considered in the first phase were incorporated in the econometric models.

Indoor and outdoor water demand uses per dwelling per billing period were modelled, both separately and combined. Water leakage was estimated as a function of network pressure and age of the dwelling and then substracted from the consumption billed, prior to estimating demand functions. Econometric results indicate that indoor demand is determined mainly by current household size and consumption in the previous period, while outdoor demand depends on the weather variables and, to a lesser degree, on current income and consumption in the same period one year before.

Supply-side effects embedded in the water charges variables may account for unexpected signs in the estimated coefficients of the service charge and, in some cases, water price. Although Two-stage Least Squares estimates

still showed wrong signs, use of cost data specific to the water utilities of Greater Victoria might change this conclusion. Errors in the measurement of the dependent variable, aggregation problems and small variability of the data on the explanatory variables are other potential sources of explanation for the wrong signs.

The estimated price and income elasticities are similar in magnitude to those obtained in other studies. The elasticities are small for indoor demands, and the income elasticity of outdoor demand is larger than the indoor one. This is a expected result, since indoor demands are of a "basic needs" nature. The long run elasticities are much larger than the short run elasticities, indicating that it takes several periods for water demand to fully adjust to changes in the explanatory variables.

The findings of this study indicate that the dynamic comprehensive model should be used for water demand forecasting in Greater Victoria. This model has good theoretical foundations, it captures most of the features of the residential water demand process, and produced reasonable estimates. The comprehensive model also performs well when forecasting demand outside the range of historical data.

The methodology developed in the two phases of this project could be tested further using more disagregated Greater Victoria data or data from other existing data banks. Some of the current unexpected econometric results might be corrected when the models are tested with the disaggregated data.

RESUME

Ce rapport présente les résultats d'une extension d'un projet antérieur (Gai 1988) d'estimation de fonctions de demande d'eau résidentielle pour les municipalitées de Victoria, Esquimalt, Oak Bay et de Saanich en Colombie-Britannique -- collectivement regroupées sous le nom du grand (Greater) Victoria. Conformément aux conclusions de la première étude, celle-ci approfondit les spécifications théoriques et économétriques du modèle développé. Un modèle linéaire et un modèle compréhensible dérivés de la théorie économique sont comparés au modèle de logarithme-logarithme (log-log) utilisé dans la première étude. Des versions statiques et dynamiques de chaque modèle sont verifiées. Les modèles dynamiques de type d'ajustements de flux et de stocks sont aussi évalués. Des variables indépendants qui n'avaient pas été examinés dans la première étude sont inclus dans les modèles économétriques de cette étude.

Les demandes d'eaux pour l'utilisation à l'intérieur et à l'extérieur de la demeure par habitation et par délai de facture sont modèlées séparément et ensemble. La perte d'eau est estimée en function de pression d'eau du réseau de distribution et de l'âge de la maison et avant d'estimer les fonctions de demandes d'eau elle est soustrait du montant d'eau consommé. Les résultats économétriques indiquent que la demande d'eau pour l'intérieur de la demeure est determinée principalement par le nombre dans la maisonnée et par la consommation d'eau de la période antérieure. Quand à la demande d'eau pour l'utilisation à l'extérieur de la demeure elle dépend de variables climatiques mais moins du revenu et de la consommation d'eau

pour la même période de l'année antécédente.

Des valeurs inattendues (soi positives ou négatives) d'estimation de coefficients de frais de service et dans certain cas de tarif d'eau sont observées.Cela peut être expliqué par les effets d'approvisionnements d'eaux faisant parti des variables de tarification. Tout de même les estimés de l'analyse Moindres Carrés de "Deux Etapes" démontrent des résultats contraires. L'utilisation de données de côuts, spécifique aux sérvices publiques d'eau du grand Victoria, pourrait changée ces observations. D'autres sources potentielles pour expliquer ces observations sont des erreurs de mesurages de variables dépendants, des problèmes d'agrégations et de petites variabilitées de données de variables indépendants.

Les estimées d'indices d'élasticitées de coûts et de revenus sont semblable à ceux d'autres études. Les indices d'élasticitées pour les demandes d'utilisation d'eau à l'intérieur des demeures sont petites mais celle de revenu pour la demande d'eau à l'extérieur est plus élevé. On s'attend à cela puisque les demandes d'eaux à l'intérieur des demeures sont de types de besoin de base. Les indices d'elasticitées de longues durées sont beaucoups plus élevés que ceux de courtes durées. Cela indique que ca prend plusieurs périodes de temps pour que la demande d'eau s'ajuste complétement aux changements des variables indépendants.

Les résultats de cette étude indiquent que le modèle dynamique compréhensible doit être utilisé pour la prévision de demande d'eau pour le

grand Victoria. Ce modèle a de bonnes bases théoriques et il comprend la majorité des aspects du procès de la demande d'eau résidentielle et il produit des estimations raisonnables. Il prévoit aussi très bien la demande d'eau hors des limites des données historiques.

La méthodologie développée dans les deux études peut être verifées d'advantage en utilisant des données désagrégées du grand Victoria ou de données d'autres banques. Quelqu'uns des résultats économétriques inattendus pourront être rectifés quand les modèles seront vérifiés avec des données désagrégées.

TABLE OF CONTENTS

ABSTRACT		i
RESUME		iii
TABLE OF CONTENTS		vi
LIST OF TABLES		viii
LIST OF FIGURES		viii
I.	INTRODUCTION	1
	A. Study Objectives	2
	B. Report Outline	2
II.	MUNICIPAL WATER SUPPLY AND DEMAND	5
III.	RESIDENTIAL WATER DEMAND MODELS	11
	A. The Nature of Residential Water Demand	11
	B. Engineering Models of Water Demand Forecasting	13
	C. Econometric Models of Water Demand Forecasting	13
	D. Simulation Models of Water Demand Forecasting	15
	E. Summary	17
IV.	ECONOMETRIC MODELS OF RESIDENTIAL WATER DEMAND	19
	A. Functional Forms of Econometric Models	19
	B. Static and Dynamic Models	23
	C. Aggregation Problems	24
	D. Modelling Time Patterns of Residential Water Demand	25
	E. Indoor, Outdoor, and Leakage Water Uses	26
	F. Specification of the Price Variable	27
	G. Supply and Demand Simultaneity	31
	H. A Comprehensive Econometric Model	33
	I. Summary	34

V. ESTIMATION OF RESIDENTIAL WATER DEMAND FUNCTIONS 35

A. Econometric Models Used 35

B. The Data 40

C. Aggregation Level 41

D. Unit Time Period 41

E. Actual Versus Recorded Water Use 42

F. Indoor and Outdoor Water Uses 43

G. Price Variables 45

H. Estimated Residential Water Demand Functions 45

I. Discussion of Econometric Results 59

J. Summary and Conclusions 63

VI. DEMAND FORECASTING AND VALUE OF WATER 65

A. Demand Forecasting 65

B. Value of Water 68

VII. CONCLUSIONS 71

REFERENCES 75

APPENDIX A A Cobb-Douglas Model of Residential Water Demand 79

APPENDIX B A Stone-Geary Model of Residential Water Demand 81

APPENDIX C A CES Model of Residential Water Demand 83

APPENDIX D A Dynamic Model of Residential Water Demand 85

APPENDIX E Aggregation and the Representative Consumer Approach 89

APPENDIX F Supply and Demand Simultaneity 91

APPENDIX G A Comprehensive Model of Residential Water Demand 97

APPENDIX H Data Appendix 103

APPENDIX I Two-Stage Least Squares. Regression Statistics 113

APPENDIX J A Comparison of Stock- and Flow-adjustment Models 119

LIST OF TABLES

1. Econometric Models of Residential Water Demand 20

2. Econometric Models 36

3. Regression Statistics 49

4. Price and Income Elasticities of Demand 53

5. Residential Water Demand Functions 54

LIST OF FIGURES

1. Consumer Demand 29

2. Specification of the Price Variables under Block Rate Structures 30

3. Victoria/Esquimalt: Actual and Predicted Water Consumption Per House 56

4. Oak Bay: Actual and Predicted Water Consumption Per House 57

5. Saanich: Actual and Predicted Water Consumption Per House 58

6. Victoria/Esquimalt: Water Demand Forecast 66

7. Victoria/Esquimalt: Value of Water (Summer 1991) 69

I. INTRODUCTION

This study is an extension of an earlier analysis of residential water demand in Greater Victoria (Gai 1988)[1]. The earlier study used readily available data to estimate residential water demand functions for the study area. Single-equation static and dynamic log-log models were used, on a per-dwelling and per-capita basis. The study found that household size and income, water charges, and summer temperature and rainfall, all contributed to determine water consumption in the study area. The estimated elasticities agreed with those obtained in similar studies conducted in other regions. The dynamic econometric model showed considerable potential for modelling residential water demand in Greater Victoria.

The earlier study utilized relatively simple theoretical and econometric models that have been used quite frequently in other studies (Howe et al. 1967, Sewell et al. 1974, Agthe and Billings 1980). The models used in estimation were not rigorously derived from the theory of consumer behaviour and as a result imposed severe restrictions on the form of the underlying utility function. Supply-side effects were not considered in the theory or estimation of the demand functions and possible simultaneity bias may have occurred in the estimated coefficients. As a result of these shortcomings, it was decided to extend the study, using more rigorous and comprehensive theoretical and econometric procedures.

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1. The study area comprises the municipalities of Victoria, Esquimalt, Oak Bay and Saanich, hereinafter referred to as Greater Victoria.

The findings of the first study suggested several approaches that could be followed in an extension of the project. These approaches included data gathering on additional explanatory variables and the use of alternative models and unit periods. Most of these approaches have been pursued in this study and are described in this report. Conclusions and the need for further data gathering and econometric work are also presented.

A. Study Objectives

The objectives of this study are to expand on the theoretical and econometric specifications of modelling residential water demand. The nature of the residential water demand and supply processes are examined in greater detail, to determine the most suitable modelling procedures. A linear model and a comprehensive econometric model, the latter derived from an underlying utility function of economic consumer theory, are to be tested against the log-log model of the first study. The performance of explanatory variables not previously considered is to be evaluated. Supply-side influences on residential water demand, working through the water charges and network pressure variables, are to be investigated.

B. Report Outline

The municipal water supply/demand process, of which residential water demand is a component, is described in Chapter II. Chapter III describes alternative ways of modelling residential water demand, such as engineering and econometric models. The possibility of modelling residential water demands through a simulation model is also advanced. Chapter IV describes several econometric models in detail, listing both their advantages and

theoretical limitations. In addition to the log-log model used in the first study, a linear model and a comprehensive economic model derived from economic theory are also examined. The competing econometric models are estimated in Chapter V using Greater Victoria data and the statistical performance of the models is compared. Chapter VI includes examples of demand forecasting and computation of willingness-to-pay and consumers-surplus using the residential water demand functions derived in Chapter V. Finally, Chapter VII lists conclusions and outlines possibilities for further data gathering and econometric modelling.

II. MUNICIPAL WATER SUPPLY AND DEMAND

This chapter describes the nature of the municipal water supply and demand process. A thorough understanding of this process is required since residential water demand is just one component of an interconnected process that involves engineering and socioeconomic considerations. Residential water demand is frequently modelled as a function of explanatory variables that are assumed to be exogenous. However, water charges are usually endogenous since they depend on the cost of water supply, which in turn depends on water demands of the residential, commercial, industrial and other user sectors. Network pressure likewise influences demand and is in turn a function of the total municipal demand. Residential water demand would therefore be better modelled in the context of a municipal supply/demand system.

Water is typically supplied by a municipal utility and distributed to residential, commercial, industrial and institutional customers. In addition, some water is used for public purposes such as parks, street flushing and fire protection. The utility obtains its water from surface or underground sources using its own facilities or, alternatively, it purchases water from other utilities. If necessary, the water is then treated and conveyed by mains to distribution reservoirs conveniently located throughout the service area. These reservoirs provide enough storage to meet a neighbourhood's expected daily consumption while guaranteeing a minimum delivery pressure, as well as providing some reserve volume for fire fighting. The water is taken from the reservoirs for

delivery to individual customers by means of a distribution network. After the water has been used by customers, the return flows are collected in the sewage network. The waste water is subjected to some level of treatment in a sewage treatment plant and is finally disposed of through ground disposal or discharge into a body of water.

The utility faces a water demand that exhibits definite time patterns. Municipal demand varies hourly and has two daily peaks, due mainly to the consumption pattern of the residential sector. Commercial and industrial water demands subside on weekends. Municipal demand also exhibits a pronounced summer peak, due mainly to the demand for lawn watering by the residential sector. The different structures of the municipal water supply system are therefore designed to meet demands of different durations. For instance, the intake works and the mains are designed to meet the average daily water demand of the population served. The distribution reservoirs are designed to meet a neighbourhood's peak daily demand as well as providing an additional reserve volume for fire fighting. The street and individual customer's lines are designed to meet peak hourly demands. Forecasting those water consumption demands is crucial for the design of planned expansions. The utilities prepare demand forecasts based on the expected growth of population, housing, and industrial activity. An additional variable, price, will also affect future water demands.

The price of water provides a link between municipal water supply and demand. The capital cost and operation and maintenance costs of the municipal water supply infrastructure are ultimately borne by the customers

of the utility. In general, the utility does not charge a single unit price for the water supplied. Instead, the utility usually has a rate structure made up of a flat-rate service charge and a consumption charge that depends on the amount of water consumed. Many municipal utilities in Canada charge only a flat-rate service charge to their residential and commercial customers (Environment Canada, 1989). The service charge varies between customer classes and generally depends on the size (pipe diameter) of the connection serving the customer. Most of the consumption charges are in the form of a declining block rate structure, i.e., the unit price charged for the first few hundred cubic metres (first block) consumed in the billing period is higher than that charged for the additional consumption beyond that threshold. Two or more blocks are used in this type of rate structure. For instance, in the case of a municipality that meters its residential customers, the first block might correspond to the normal consumption of residential customers, the second to that of commercial and light industrial users and the third to the consumption of heavy industrial customers. Economies of scale in the provision of water are cited in support of this rate structure. A few utilities charge the same unit price regardless of the consumption level. Others impose an increasing block rate structure.

The rate structure conveys important information for the customers' decision making process. For instance, residential customers periodically update their stock of water-using appliances based on long term trends of family income and size, climate and rate structure; all economic variables considered are adjusted for inflation. In a given billing period, these

customers select their water consumption level (rate of use of existing stock) based on family size, economic, and weather conditions. Similarly, industrial customers adopt industrial processes and adjust their stock of equipment based on expected long term demand for their industrial output and on trends of input costs, water being one of them. The level of industrial output that influences the demand for industrial water depends on overall economic conditions and on production versus inventory decisions. The actual level of industrial water consumption in a given period will be determined by the input/output relationship inherent in the existing equipment stock.

As the municipality and related industry expand, more intake capacity and/or new sources of water will be needed. Expansion of the capacity of treatment plants and of the capacity and coverage of the distribution network will also be required. Once the economies of scale are exhausted, the average and marginal cost of water supply will increase and the utility will have to charge a higher water price. The response of customers to price changes - the price elasticity of demand - is important for the utility's system planning. If the customers reduce their consumption in response to a higher price in a significant way, the planned capacity expansions may not be immediately required. The utility, therefore, may deliberately delay expensive capacity additions by using price as a tool to control the demand growth.

The price elasticity of demand will differ across different customer classes, depending on the alternative sources of supply available to each

category and on the value customers in each class attach to consumption. For instance, large industrial customers might find it more economical in the long run to supply their own water rather than to purchase it from the utility. On the other hand, even if they continue to get their supply from the utility, they may adjust their industrial processes to conserve or recirculate the water purchased. Residential customers usually find it cheaper and more dependable to buy water from the utility than to rely on alternative forms of self supply. They will normally respond to a higher price of water by reducing the consumption of more superfluous uses such as lawn watering. The actual price elasticity of demand for the different customer classes has to be determined by empirical studies such as this one.

This study has dealt with only one component of the municipal water supply/demand process described above - the demand for water by the residential sector. The water demand of the commercial, industrial and other sectors has not been studied here, nor the interaction of the demand of all sectors with the supplier (the utility). This report, therefore, is mainly concerned with modelling residential water demands. However, supply side influences on residential water demand working through the water charge variables, have been allowed for in this study.

III. RESIDENTIAL WATER DEMAND MODELS

A log-log econometric model was used in Gai (1988) to derive the initial residential water demand functions. As a result of the study findings, an investigation of additional econometric models was suggested and forms the basis for this study. This chapter provides an overall description of econometric and other approaches for modelling residential water demand.

Historically, water demand studies have used the so called engineering approach in which economic variables, such as price, have not been considered until very recently. An alternative approach, using an econometric model, does incorporate price as a variable influencing water demand. A further approach, the use of a simulation model, is discussed here since the complexity of the problem at hand leads to models that do not always have a convenient algebraic solution, as econometric models usually assume. As a background for a review of the alternative modelling procedures, a brief description of the residential water demand process is presented.

A. The Nature of Residential Water Demand

The basic unit of the residential water demand sector is the household. Each household is equipped with a stock of water-using appliances and, given the number of people in the dwelling and their income, the household will respond to weather and price variations and select a water consumption level and its allocation among several indoor (drinking, cooking, bathing, laundry) and outdoor uses (lawn watering, car washing, swimming pool

filling). Significant outdoor uses influenced by summer weather conditions will occur mainly for houses but not for apartments. Unvanted consumption in the form of leakages will occur as a result of deteriorated plumbing, a situation which will worsen if the network's pressure is high.

The household's stock of water-using appliances is adjusted over time based on long term trends of household's income and size, the price of water and other goods, and the price of water-using appliances. Depending on technical changes in society, there may also be adjustments in water consumption due to changes in long term trends of consumption habits and the efficiency of water-using appliances.

Households are therefore engaged in a 2-stage decision making process:

1. They occasionally update the stock of water-using appliances.
2. They continuously select a water consumption level (the rate of use of the current stock) and its allocation to indoor and outdoor uses.

The sum of the water consumption of the individual households determines the residential sector's water consumption in a municipality. The levels of water consumption, for a given time period, vary across households reflecting their different socioeconomic characteristics and preferences. In addition, water consumption for a given household, or for the whole residential sector, also varies over time on an hourly, daily, weekly, monthly and yearly basis. The utility is interested in forecasting water demands on a municipal, neighbourhood and other spatial basis. The utility determines peak and average demands for the relevant area of service on an

hourly, daily and seasonal basis. As previously discussed, the utility uses "forecasts" to design different structures for the system's expansion. Alternative methods of demand forecasting are described in following sections of this chapter.

B. Engineering Models of Water Demand Forecasting

Water demand forecasting in the traditional engineering approach is done by multiplying forecasted population and housing growth by historical water use rates, and adding an allowance for an increased standard of living (Howe and Linaweaver 1967, Davis and Sorensen 1969). This type of water demand forecast constitutes a "requirement" that must be met, regardless of cost. Economic theory, upon which econometric models are based, suggests that water demand will respond to price and that the connection between supply and demand through the cost of water supply and price should be taken into account in system planning.

The forecasting of seasonal, weekly, daily and hourly patterns of demands in the engineering approach is based on a consideration of historical patterns and planned increases in storage. No allowance is usually made for a potential shifting of time patterns induced by a change in a rate structure.

C. Econometric Models of Water Demand Forecasting

An analysis of the indoor, outdoor and leakage uses to which water is allocated in a typical household suggests a list of potential explanatory variables that contribute to determining water consumption levels (Gai

1988). For instance, water consumption for indoor and outdoor uses increases with family size and income. Outdoor water consumption increases in hot and dry summers. Water wastage through leakages in the plumbing increases with the age of the dwelling and with the water pressure in the distribution network. Indoor and outdoor water uses decrease with higher water prices; less influence on indoor uses since they tend to satisfy basic needs.

An econometric model formalizes the above intuitive process by postulating a specific functional form for the relationship between the household's water consumption level and the explanatory variables (family size and income, price of water, price of other goods, summer weather variables, age of dwelling, network's pressure, and others). The functional form can be of an ad-hoc nature (e.g. linear), or it can be derived from the solution of an underlying economic model of consumer demand.

Econometric models require data on the dependent variable (water consumption) and on the independent (explanatory) variables for estimation of water demand. Water consumption by individual households is recorded once in every billing period in those municipalities that have installed meters for each individual connection. Most billing periods are in the order of two to four months; therefore, consumption breakdowns on a monthly, weekly, daily or hourly basis are not available. The data on the explanatory variables are reported with different degrees of frequency and aggregation level, depending on the variable:

1. Population and dwelling data are reported every five years and

sometimes annually.

2. Income is reported annually for the whole municipality but not for individual households.
3. The rate structure is available on a continuing basis.
4. Relevant weather variables are reported on a daily and sometimes hourly basis for one or more weather stations in the municipality.
5. Data on some explanatory variables are not recorded or are difficult to obtain (e.g. stock of water using appliances) and they must be proxied by other variables or else left out of the econometric model.

A comprehensive data base for a selected area or sample of households can be gathered through continuous monitoring of water consumption supplemented by the use of questionnaires to obtain data on the explanatory variables.

The aggregation level and unit time period used in econometric models are generally determined by data availability. Most models use data aggregated at the municipal or neighbourhood level. Models that use time series data adopt a unit time period of a year, a season (winter, summer), or a billing period (e.g. a month or a four-month period). Economic considerations tend to support the choice of the household as the water demand decision unit and the billing period as the time period. In addition, a billing period of about four months tends to coincide better with seasonal consumption patterns induced by weather conditions.

D. Simulation Models of Water Demand Forecasting

An econometric model is basically an algebraic equation that is the

solution of the consumer's optimization problem, which is to select consumption levels of different goods in order to maximize well being, subject to budget limitations. The consumer's preferences are represented by an utility function whose arguments are the consumption levels of the different goods. As it will become clear in Chapter IV, and given the complexity of the actual problem, restrictions must be sometimes imposed on the form of the utility function in order to obtain a convenient algebraic solution to the model.

In order to keep the form of the underlying utility function as unrestricted as possible, the problem can be modelled using a simulation approach. A fairly general utility function can be postulated and initial values of its parameters can be assumed. The consumer's optimization problem can then be solved for each observation using an algorithm and the applicable values of the explanatory variables. The water consumption value that solves the optimization problem will be different from the actual recorded value for that particular observation and the corresponding error can be computed. The sum of squared errors can be computed by repeating the procedure for all the observations, The overall solution to the problem can be obtained by finding a set of parameters for the utility function that minimize the sum of squared errors. This can be done by using a grid search procedure.

The greater the number of parameters and observations, the more expensive the grid search procedure. On the other hand, a simulation approach might be particularly well suited to solve an overall municipal water

supply/demand model that considers water demands of all sectors and allows for the discrete sizes of the supply system expansions.

E. Summary

This chapter has described the nature of the residential water demand process and briefly discussed engineering, econometric and simulation models of water demand. Econometric models were selected in favour of engineering models to model residential water demand in the study area because econometric models account for the role of income and prices. Residential water demand could also be modelled as part of a simulation model of municipal water supply and demand, given the interrelated nature of the municipal water supply and demand process. Current data are not sufficient to develop a simulation model for the study area. A detailed description of the econometric models used in this study are discussed in the following chapter.

IV. ECONOMETRIC MODELS OF RESIDENTIAL WATER DEMAND

Previous work on residential water demand (Gai 1988) used standard but restrictive econometric models of water demand. This chapter outlines a more comprehensive approach to the problem, describing a range of econometric models of residential water demand used to derive the water demand functions presented in Chapter V. A linear model and a comprehensive model were used in addition to the log-log model used in Gai (1988). Static and dynamic versions of each model were studied. A number of special characteristics of residential water demand and the way they are accounted for in the different models are discussed. The theoretical basis of each model and their limitations to reflect all aspects of water demand are also explained. The comprehensive econometric model introduced in this chapter is derived from economic theory and accounts for most of the characteristics of residential water demand.

A. Functional Forms of Econometric Models

The functional form of an econometric model can be adopted based on reasons of algebraic convenience, or it can be derived from an underlying economic model. Models 1 and 2 in Table 1 have a linear form and a log-log form respectively and are examples of the first approach. As shown below, these convenient functional forms cannot, in general, be derived from an economic model of consumer's utility, even when fairly strong restrictions are imposed on the form of the underlying utility function. Despite these objections, models 1 and 2 have been frequently used in previous studies (Howe and Linaweaver 1967, Agthe and Billings 1980, Gai 1988). Their

Table 1. Econometric Models of Residential Water Demand

1. **MODEL 1 (LINEAR)**

A. Static Model

$$w_t = A^1 + B^1 * n_t + C^1 * Y_t/q_t + D^1 * SC_t/q_t + E^1 * p_t/q_t + F^1 * PE_t + G^1 * R_t + \varepsilon^1{}_t$$

B. Dynamic Model

$$w_t = A^1 + B^1 * (n_t + n_{t-1}) + C^1 * (Y_t/q_t + Y_{t-1}/q_{t-1}) + D^1 * (SC_t/q_t + SC_{t-1}/q_{t-1}) + E^1 * (p_t/q_t + p_{t-1}/q_{t-1}) + F^1 * (PE_t + PE_{t-1}) + G^1 * (R_t + R_{t-1}) + H^1 * w_{t-1} + \varepsilon^1{}_t$$

2. **MODEL 2 (LOG-LOG)**

A. Static Model

$$\log(w_t) = \log(A^2) + B^2 * \log(n_t) + C^2 * \log(Y_t/q_t) + D^2 * \log(SC_t/q_t) + E^2 * \log(p_t/q_t) + F^2 * \log(PE_t) + G^2 * \log(R_t) + \upsilon^2{}_t$$

B. Dynamic Model

$$\log(w_t) = \log(A^2) + B^2 * [\log(n_t) + \log(n_{t-1})] + C^2 * [\log(Y_t/q_t) + \log(Y_{t-1}/q_{t-1})] + D^2 * [\log(SC_t/q_t) + \log(SC_{t-1}/q_{t-1})] + E^2 * [\log(p_t/q_t) + \log(p_{t-1}/q_{t-1})] + F^2 * [\log(PE_t) + \log(PE_{t-1})] + G^2 * [\log(R_t) + \log(R_{t-1})] + H^2 * \log(w_{t-1}) + \upsilon^2{}_t$$

3. **MODEL 3 (COMPREHENSIVE)**

A. Static Model

$$w_t = A^3 + B^3 * n_t + C^3 * Y_t/p_t + D^3 * SC_t/p_t + E^3 * PE_t + F^3 * R_t + G^3 * n_t * q_t/p_t + \varepsilon^3{}_t$$

B. Dynamic Model

$$w_t = A^3 + B^3 * (n_t + n_{t-1}) + C^3 * (Y_t/p_t + Y_{t-1}/p_{t-1}) + D^3 * (SC_t/p_t + SC_{t-1}/p_{t-1}) + E^3 * (PE_t + PE_{t-1}) + F^3 * (R_t + R_{t-1}) + G^3 * (n_t * q_t/p_t + n_{t-1} * q_{t-1}/p_{t-1}) + H^3 * w_{t-1} + \varepsilon^3{}_t$$

Continued.

Table 1. Concluded.

NOTATION

w : household water consumption
n : household size (number of people)
Y : household disposable income
SC : service charge
p : marginal price of water
q : price of composite good = consumer price index
PE : summer potential evapotranspiration
R : summer rainfall
ε : error term
υ : error term

The superscripts 1, 2 and 3 refer to the corresponding model and the subscripts t and t-1 refer to the current and previous period respectively.

The parameters $A^1,B^1,\ldots, A^2,B^2,\ldots, A^3,B^3,\ldots,F^3$ are estimated by Least squares (LS) or Maximum Likelihood (ML) methods.

The comprehensive model is based on a Stone-Geary utility function (Appendix G):

$$U = K*n*[(w+R-PE)/n-wo]^{a}(z/n-zo)^{b}$$

The parameters a,b,w_o and z_o can be estimated as follows:

$$\hat{a}^2/\hat{b}^2 = \hat{B}^2/\hat{D}^2 \qquad \hat{w}_o = \hat{A}^2/\hat{D}^2 \qquad \hat{z}_o = \hat{F}^2/\hat{B}^2$$

where the $\hat{}$ symbol denotes an estimate of a parameter obtained by the LS or ML methods.

simplicity have been used to provide estimates of price and income elasticities of demand.

The economic theory of consumer demand assumes that the preferences of the consumer can be represented by a utility function whose arguments are the consumption levels of the different goods. The utility level of the consumer increases as his consumption levels increase. The problem for the consumer can be formally stated as one of selecting the consumption levels of the different goods that maximize the consumer's utility level, given the price of the goods and services and subject to a budget constraint.

The derivation of demand functions for a typical consumer in a simple economy with only two goods is illustrated in Appendix A. The demand for a good such as water can be studied, under certain conditions, as a problem of allocating a consumer's budget between only two goods, water and a composite good that represents all other goods and services (Varian 1984). The Cobb-Douglas utility function in the model of Appendix A includes weather variables as additional arguments since water demand depends on summer weather conditions. Similar models are derived in Appendixes B and C using a Stone-Geary and a Constant Elasticity of Substitution (CES) utility function. The Stone-Geary utility function seems particularly suited to study residential water demand since it allows for minimum (subsistence) levels of consumption that are met first, leaving the remainder of the budget to be allocated between additional amounts of water and composite goods and services that meet less basic needs.

The water demand functions derived from the models of Appendixes A, B and C are not linear (or linear in logs) in income and prices. It is unlikely that the linear and log-log models can be derived from any known utility function. Furthermore, residential water demands present additional complications that increase the complexity of the models of Appendixes A, B and C. It is even less likely that more complex models will generate simple linear or log-log solutions.

B. Static and Dynamic Models

The econometric models discussed so far in this chapter assume that residential water consumption adjusts instantaneously to changes in the levels of the explanatory variables. However, it might take a significant period for water consumption to fully adjust to a price change. Three main explanations can be given in support of using dynamic models of residential water demand to account for lagged adjustments in water consumption. First, there are water using habits that take a while to modify. Second, changes in the explanatory variables might be perceived as temporary until it becomes obvious that the changes are permanent. Third, as Agthe and Billings (1980) p. 477 state, "water use is determined by a stock of water-using consumer durables....Thus, changes in water price or other independent variables may immediately change the rate of use of the stock of durable goods, but the stock of water-using consuming durables itself will change gradually over an extended period of time".

Linear dynamic models of residential water demands of the flow-adjustment and stock-adjustment types are derived in Appendix D. In flow-adjustment

models, water consumption is assumed to adjust partially towards a desired level of consumption which is a function of the exogenous variables. In the stock-adjustment approach, the stock of water-using appliances is partially adjusted towards a desired optimal level which is a function of both the expected future values of the exogenous variables and the price of the stock. The actual water consumption level in a given period is a function of the current levels of the exogenous variables and stock of water-using appliances.

The two models of Appendix D capture the main features of all dynamic models, namely:

1. The solution includes lagged values of the independent variables.
2. The solution includes the lagged value of the dependent variable.
3. The solution includes the price of the stock of water-using appliances, in stock-adjustment models.

C. Aggregation Problems

The water consumption of the residential sector represents the aggregation of the consumption of thousands of individual connections. Each household has different socioeconomic characteristics and preferences, and water consumption across households differs accordingly. However, econometric studies often have to use municipal average data since consumption, income, and other data are usually not available for individual households. In other words, all households are assumed to be identical. The socioeconomic characteristics of the representative household are, by assumption, equal to the municipal averages, and the preferences of the representative

household are such as to generate a water consumption level equal to the average residential consumption in the municipality.

Economic theory requires that individual preferences meet a strong set of conditions for the representative consumer (household) approach to be valid. One of the conditions is that the marginal utility of income be the same for each household. Appendix E derives such conditions for the models of Appendixes A, B and C.

There is not much that can be done to address the aggregation issue when only municipal average data are available, other than to acknowledge that an aggregation problem may exist. One of the interesting and important implications of this problem is that, should individual data be available, individual demands might be found to be price-elastic and aggregated demand price-inelastic. Such a finding would be important for the utility, since the utility is concerned with aggregate demands, at the neighbourhood and municipal levels. Residential water demand could be found to be price-inelastic at the municipal level but price-elastic in wealthy neighbourhoods.

D. Modelling Time Patterns of Residential Water Demand

Aggregation problems can also occur on a temporal dimension. Residential water consumption varies hourly, presenting two mild daily peaks in winter (morning, early evening), and one very pronounced peak in summer (early evening), the latter due to lawn watering. Only indoor uses tend to occur in winter and, as a result, residential water consumption is lower and

fairly constant in winter compared to summer. The indoor water consumption does not markedly change in summer but outdoor uses become highly significant and variable in response to weather conditions. These general patterns apply at the household as well as at the municipal level.

Water consumption data for each household are recorded once in a billing period; this is typically done every two to four months. There is no record of weekly or daily variations since the available reading represents the average consumption over the period. As a result, econometric studies that use daily or weekly consumption as the dependent variable are not frequently conducted. As the unit time period increases from one month to a few months, the seasonal variations in water consumption are captured. Econometric models that are forced to use average annual consumption data disregard most of the winter to summer variation. More important, outdoor water uses do not occur throughout the year and, as discussed below, indoor and outdoor water use can be considered as two different economic goods. Residential water consumption can be better modelled if the unit period matches the seasonal patterns of indoor and outdoor consumption.

E. Indoor, Outdoor, and Leakage Water Uses

As mentioned in Chapter III, a typical household allocates its water consumption amongst several indoor and outdoor uses. There is also a third category, unwanted use, due to leakages through the dwelling's plumbing. A fourth type of "use", positive or negative, occurs if the meter is over or underrecording.

In the short run, the household cannot control the leakage which is mainly determined by pressure in the distribution network and the condition of the plumbing, the latter usually proxied by the dwelling's age. In the long run, if water charges are high enough, the household has an incentive to repair leaks and to request a meter test.

Individual meters are read once in every billing period and the reading gives the sum of the four types of uses (indoor, outdoor, leakages, meter malfunctioning). Separate readings for each category are rarely available and the breakdown must be done based on sensible a priori assumptions. It is often assumed that leakage and meter problems are not significant.

From the consumers' point of view, indoor and outdoor water uses are different economic goods. Indoor uses (drinking and cooking, bathing, laundry) generally meet basic needs and occur all year round. Outdoor uses (lawn watering, car washing, swiming pool filling) are less necessary and tend to occur mostly in summer. Furthermore, outdoor uses occur mainly in houses since they are considerably reduced in apartments. One would expect that indoor uses are more price and income inelastic than outdoor ones, since they satisfy more basic needs.

F. Specification of the Price Variable

Water, like other goods such as electricity, is not always priced at a uniform price and the price varies with the amount consumed. Many Canadian municipal water utilities do not meter their residential customers and impose a flat rate charge. Decreasing block rate structures prevail among

those utilities whose residential customers are metered. Some municipalities charge the same unit price regardless of the amount consumed, or use an increasing block rate structure. Figure 1 illustrates the consumer's problem for a good with constant unit price, with and without a service charge. Water is often subject to a fixed charge in addition to a consumption charge. Figure 2 expands on the subject by analyzing declining and increasing block rate structures. It can be shown (Taylor 1975, Nordin 1976) that cases of service charges and block rate structures can be made equivalent to the uniform price case by using the following two price variables instead of one, as shown in Figures 1 and 2:

1. The **marginal price** that applies on the block of the rate structure corresponding to the actual consumption (price OA for consumption OF in Figures 1 and 2).
2. A **difference variable**, defined in Agthe and Billings (1980) as the "difference between what the typical consumer actually pays for water and what would be paid if all water were purchased at the marginal rate" (distance M-M" in Figures 1 and 2). This second variable summarizes the contributions of preceding or inframarginal rates and charges and is equivalent to an implicit tax or subsidy associated with decreasing or increasing rate schedules.

Block rate structures create an econometric problem, "simultaneity", since two independent variables - marginal price and the difference variable - are a function of consumption, the dependent variable.[2] Therefore, the

2. The use of an alternative variable, average price, computed as service charge plus consumption charge and divided by consumption, would create the same type of simultaneity problem.

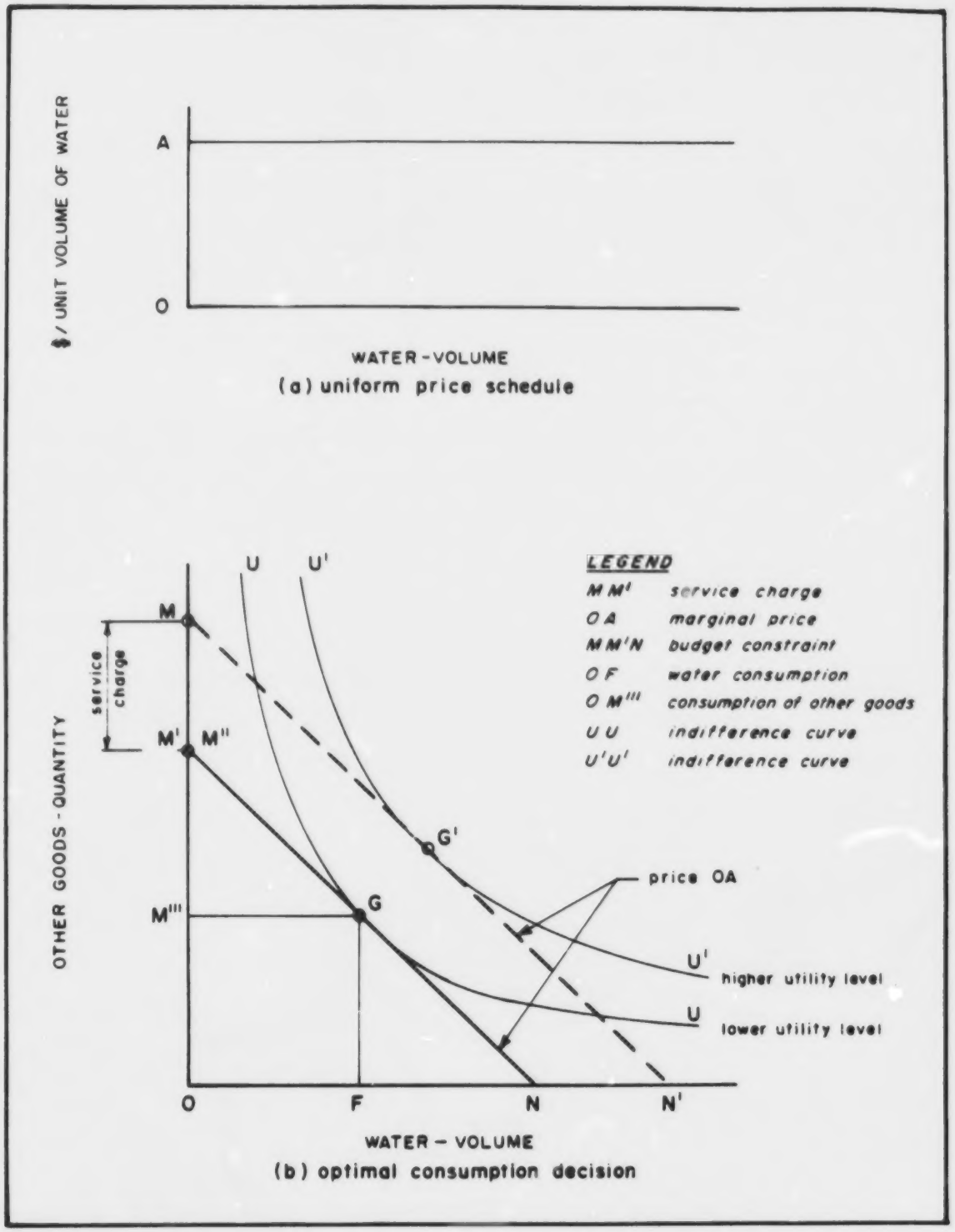

Figure 1 Consumer Demand

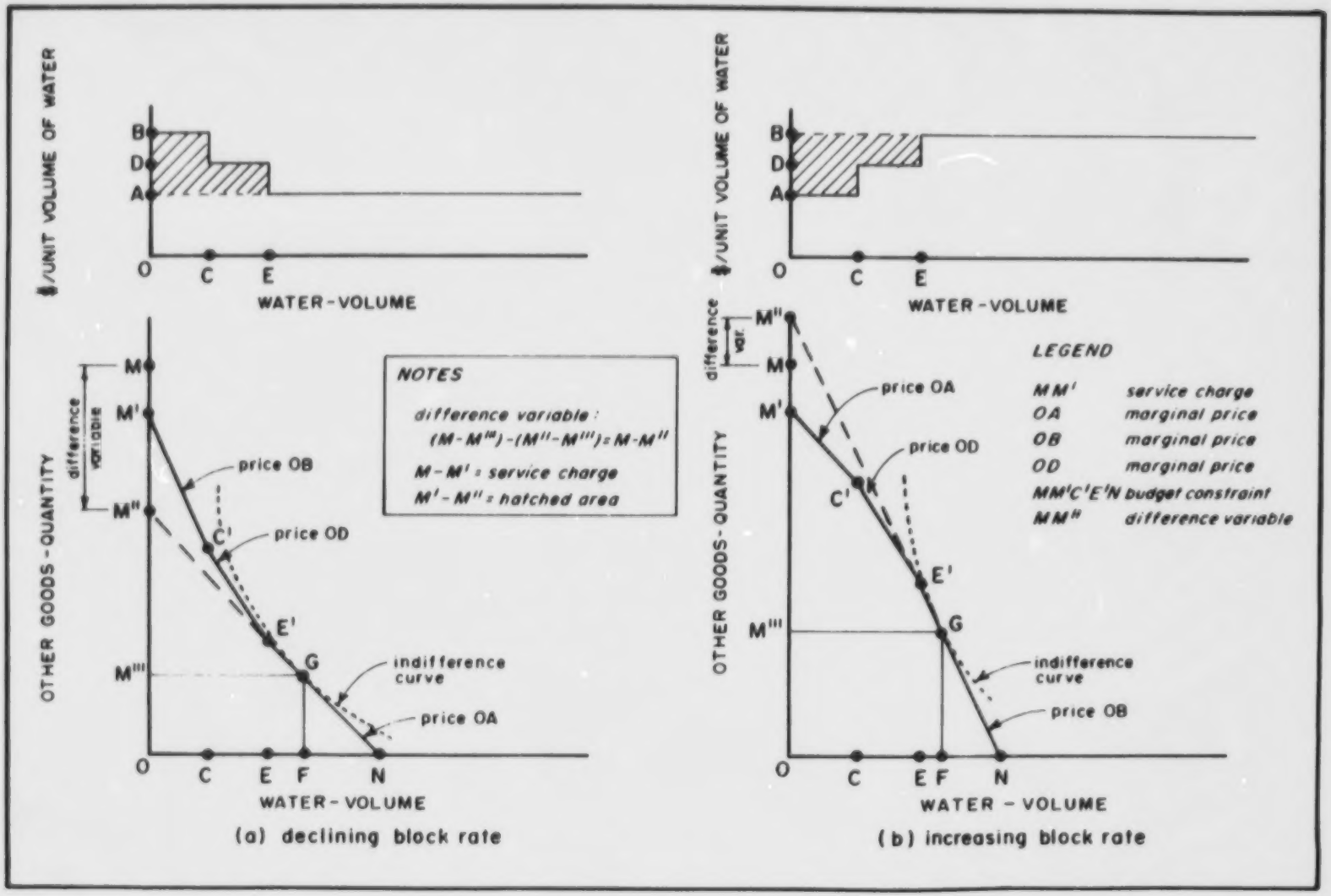

Figure 2 Specification of the Price Variables under Block Rate Structures

independent variables occur on both sides of the regression equation. The simultaneity problem results in biased estimates of the coefficients of the regression. An instrumental variable method can be used to handle this simultaneity problem (Jones and Morris 1984). The instrumental variables procedure, as stated in Renzetti (1985), "involves replacing ... marginal price with an estimate derived by regressing it against a set of 'instruments', i.e. a set of variables highly correlated with the (marginal price) but (uncorrelated with) the error term (of the original regression). It can be shown that this procedure yields consistent, unbiased parameter estimates .... The common problem underlying this approach is to choose the instrumental variable correctly".

G. Supply and Demand Simultaneity

The price variables appear on the right hand side of the water demand functions while household water consumption appears on the left hand side. As shown below, the price variables may be a function of household consumption as a result of the pricing policy of the utility. If this is the case, a simultaneity problem exists that will lead to biased estimates.

The service charge and price structure are set by the utility to recover the fixed and variable costs associated with supplying water. The fixed costs represent mainly the capital cost of the supply system and the administrative costs of metering and billing. The variable costs consist mainly of water purchases from the water district, the cost of pumping water into the neighbourhood reservoirs, and the cost of connecting new customers to the system.

The capacity of the supply system is based on some expected future municipal consumption plus a reserve margin. The capital cost of the supply system is a function of the corresponding consumption per connection and of the total number of connections. The administrative costs are a function of the number of connections. Total variable costs depend on the actual consumption per connection and on the number of connections.

If the utility recovers its fixed costs through the service charge and its variable costs through average cost pricing, then the service charge will be a function of household consumption, whereas price will not be a function of such consumption (Appendix F). A simultaneity problem exists in the case of the service charge variable and this will lead to biased estimates. Alternative pricing rules may make price, or both service charge and price, a function of household consumption (Appendix F). This simultaneity problem is independent of the simultaneity problem introduced by block rate structures.

Unbiased estimates can be obtained using the Two-stage Least Squares method. The right hand side endogenous variables (service charge, price) are expressed as functions of supply-side variables (cost variables, population served). A system of equations consisting of the demand equation and the supply equations (service charge, price) is thus created. Service charge and price are then regressed separately on all the exogenous variables in the system and the estimates replaced in the demand equation for the final estimation.

The pressure variable also introduces a simultaneity problem as illustrated in the simple model of Appendix F. The pressure in the system drops in response to higher municipal consumption. The pressure drop reduces household leakage and indoor and outdoor consumption. The lower household consumption reduces residential consumption, and therefore municipal consumption, thus increasing system pressure. Pressure affects consumption and consumption influences pressure.

This simultaneity problem should also be handled by estimating a system of supply and demand equations. The supply equation, in this case, would express pressure as a function of appropriate supply-side variables.

H. A Comprehensive Econometric Model

As a result of the special characteristics of residential water demand and the limitations of the various methods to reflect all those aspects, a comprehensive econometric model was developed as an alternative to the standard linear and log-log models. The comprehensive model is derived in Appendix G and summarized in Table 1. The model is based on the following assumptions:

1. Aggregation level can be adopted at the individual household, neighbourhood, or municipal level, depending on available data.
2. Unit time period adopted is a 4-month period, to match both the billing period and the seasonal patterns of indoor and outdoor water consumption.
3. Meters are assumed to function correctly. Leakage is an assumed

function of pressure and dwelling's age that applies both to houses and apartments. Outdoor uses do not occur in winter months. Separation between indoor and outdoor water use is based on winter consumption patterns. Outdoor uses do not occur in apartments in summer.

4. Functional form is derived from an underlying utility function of the Stone-Geary type. The household allocates its budget into three goods: water for indoor uses, water for outdoor uses, and a composite good that represents all other goods and services.
5. The econometric model is a dynamic one; partial flow or stock adjustment type to be decided by statistical performance.
6. Price variables are marginal price and difference variable.

I. Summary

Lack of theoretical foundations of the standard linear and log-log demand models are of major concern and motivated the derivation of a comprehensive econometric model based on economic theory. The three models were designed to account for several aspects of residential water demand. Those aspects include short and long run adjustments, spatial and temporal aggregation, seasonal variations, indoor and outdoor water uses and leakage, and simultaneity problems introduced by block-rate structures and service charges, water pricing policies, and network pressure. The statistical performance of the linear, log-log and comprehensive models is evaluated in Chapter V, using Greater Victoria data.

V. ESTIMATION OF RESIDENTIAL WATER DEMAND FUNCTIONS

This chapter presents the results of econometric estimations carried out using the models described in Chapter IV. Static and dynamic versions of each model were tested. The data used were obtained from the municipalities of Greater Victoria and are extensively described in Gai (1988). Data on additional variables used in this study are described below. The econometric results are discussed in Section F of this chapter.

A. Econometric Models Used

Table 2 lists the following econometric models used for estimation:

1.S Linear, static.

1.D Linear, dynamic.

2.S Log-log, static.

2.D Log-log, dynamic.

3.S Comprehensive, static.

3.D Comprehensive, dynamic.

The linear and log-log models were considered because they provide a useful frame of reference for comparing the results of this study with those of previous studies. The comprehensive model has been added as an alternative because of its better theoretical foundations.

The six models are listed in detail in Table 2. Indoor and outdoor water demands are identified separately. In addition, the lag structure of the outdoor water demand dynamic models is explicitly identified.

Table 2. Econometric Models

1.S Linear, static

- separate modelling of indoor and outdoor water demands

$$w^i_t = A^i + B^i * n_t + C^i(Y_t/q_t) + D^i(SC_t/q_t) + E^i(p_t/q_t) + \varepsilon^i_t$$

$$w^o_t = A^o + B^o * n_t + C^o(Y_t/q_t) + D^o(SC_t/q_t) + E^o(p_t/q_t) + F^o * PE_t + G^o * R_t + \varepsilon^o_t$$

- combined modelling of indoor and outdoor water demands

$$w_t = w^i_t + w^o_t = A + B * n_t + C * (Y_t/q_t) + D * (SC_t/q_t) + E * (p_t/q_t) + F * PE_t + G * R_t + \varepsilon_t$$

1.D Linear, dynamic

- separate modelling of indoor and outdoor water demands

$$w^i_t = A^i + B^i * [(n_t + n_{t-1} + n_{t-2})/3] + C^i * [(Y_t/q_t + Y_{t-1}/q_{t-1} + Y_{t-2}/q_{t-2})/3] +$$
$$D^i * [(SC_t/q_t + SC_{t-1}/q_{t-1} + SC_{t-2}/q_{t-2})/3] + E^i * [(p_t/q_t + p_{t-1}/q_{t-1} + p_{t-2}/q_{t-2})/3] +$$
$$H^i * w^i_{t-1} + \varepsilon^i_t$$

$$w^o_t = A^o + B^o * [(n_t + n_{t-1} + n_{t-2})/3] + C^o * [(Y_t/q_t + Y_{t-1}/q_{t-1} + Y_{t-2}/q_{t-2})/3] +$$
$$D^o * [(SC_t/q_t + SC_{t-1}/q_{t-1} + SC_{t-2}/q_{t-2})/3] + E^o * [(p_t/q_t + p_{t-1}/q_{t-1} + p_{t-2}/q_{t-2})/3] +$$
$$F^o * PE_t + G^o * R_t + I^o * w^o_{t-3} + \varepsilon^o_t$$

- combined modelling of indoor and outdoor water demands

$$w_t = A + B * [(n_t + n_{t-1} + n_{t-2})/3] + C * [(Y_t/q_t + Y_{t-1}/q_{t-1} + Y_{t-2}/q_{t-2})/3] +$$
$$D * [(SC_t/q_t + SC_{t-1}/q_{t-1} + SC_{t-2}/q_{t-2})/3] + E(p_t/q_t + p_{t-1}/q_{t-1} + p_{t-2}/q_{t-2})/3] +$$
$$F * PE_t + G * R_t + H * w_{t-1} + I * w_{t-3} + \varepsilon_t$$

Continued.

Table 2. Continued.

2.S Log-log, static

- separate modelling of indoor and outdoor water demands

$$\log(w^i_t) = \log(A^i) + B^i*\log(n_t) + C^i*\log(Y_t/q_t) + D^i*\log(SC_t/q_t) + E^i*\log(p_t/q_t) + v^i_t$$

$$\log(w^o_t) = \log(A^o) + B^o*\log(n_t) + C^o*\log(Y_t/q_t) + D^o*\log(SC_t/q_t) + E^o*\log(p_t/q_t) + F^o*\log(PE_t) + G^o*\log(R_t) + v^o_t$$

- combined modelling of indoor and outdoor water demands

$$\log(w_t) = \log(A) + B*\log(n_t) + C*\log(Y_t/q_t) + D*\log(SC_t/q_t) + E*\log(p_t/q_t) + F*\log(PE_t) + G*\log(R_t) + v_t$$

2.D Log-log, dynamic

- separate modelling of indoor and outdoor water uses

$$\log(w^i_t) = \log(A^i) + B^i*\{[\log(n_t)+\log(n_{t-1})+\log(n_{t-2})]/3\} + C^i*\{[\log(Y_t/q_t)+\log(Y_{t-1}/q_{t-1})+\log(Y_{t-2}/q_{t-2})]/3\} + D^i*\{[\log(SC_t/q_t)+\log(SC_{t-1}/q_{t-1})+\log(SC_{t-2}/q_{t-2}]/3\} + E^i*\{[\log(p_t/q_t)+\log(p_{t-1}/q_{t-1})+\log(p_{t-2}/q_{t-2})]/3\} + H^i*\log(w^i_{t-1}) + v^i_t$$

$$\log(w^o_t) = \log(A^o) + B^o*\{[\log(n_t)+\log(n_{t-1})+\log(n_{t-2})]/3\} + C^o*[\log(Y_t/q_t)+\log(Y_{t-1}/q_{t-1})+\log(Y_{t-2}/q_{t-2})]/3\} + D^o*\{[\log(SC_t/q_t)+\log(SC_{t-1}/q_{t-1})+\log(SC_{t-2}/q_{t-2})]/3\} + E^o*\{[\log(p_t/q_t)+\log(p_{t-1}/q_{t-1})+\log(p_{t-2}/q_{t-2})]/3\} + F^o*\log(PE_t) + G^o*\log(R_t) + I^o*\log(w^o_{t-3}) + v^o_t$$

Continued.

Table 2. Continued.

- combined modelling of indoor and outdoor water uses

$$\log(w_t) = \log(A) + B*\{[\log(n_t)+\log(n_{t-1})+\log(n_{t-2})]/3\} + C*\{[\log(Y_t/q_t)+\log(Y_{t-1}/q_{t-1})+ \log(Y_{t-2}/q_{t-2})]/3\} + D*\{[\log(SC_t/q_t)+\log(SC_{t-1}/q_{t-1})+\log(SC_{t-2}/q_{t-2})]/3\} + E*\{[(\log(p_t/q_t)+\log(p_{t-1}/q_{t-1})+\log(p_{t-2}/q_{t-2})]/3\} + F*\log(PE_t) + G*\log(R_t) + H*\log(w_{t-1}) + I*\log(w_{t-3}) + \upsilon_t$$

3.S Comprehensive, static

- separate modelling of indoor and outdoor water demands

$$w^i_t = A^i + B^i*n_t + C^i*(Y_t/p_t) + D^i*(SC_t/p_t) + E^i*n_t*(q_t/p_t) + \varepsilon^i_t$$

$$w^o_t = A^o + B^o*n_t + C^o*(Y_t/p_t) + D^o*(SC_t/p_t) + E^o*n_t*(q_t/p_t) + F^o*PE_t + G^o*R_t + \varepsilon^o_t$$

- combined modelling of indoor and outdoor water demands

$$w_t = A + B*n_t + C*(Y_t/p_t) + D*(SC_t/p_t) + E*n_t*(q_t/p_t) + F*PE_t + G*R_t + \varepsilon_t$$

3.D Comprehensive, dynamic

- separate modelling of indoor and outdoor water demands

$$w^i_t = A^i + B^i*[(n_t+n_{t-1}+n_{t-2})/3] + C^i*[(Y_t/p_t+Y_{t-1}/p_{t-1}+Y_{t-2}/p_{t-2})/3] + D^i*[(SC_t/p_t+SC_{t-1}/p_{t-1}+ SC_{t-2}/p_{t-2})/3] + E^i*[(n_t*q_t/p_t+n_{t-1}*q_{t-1}/p_{t-1}+n_{t-2}*q_{t-2}/p_{t-2})/3] + H^i*w^i_{t-1} + \varepsilon^i_t$$

Continued.

Table 2. Concluded.

$$w^{o}_{t} = A^{o} + B^{o}*[(n_t+n_{t-1}+n_{t-2})/3] + C^{o}*[(Y_t/p_t+Y_{t-1}/p_{t-1}+Y_{t-2}/p_{t-2})/3] + D^{o}*[(SC_t/p_t+SC_{t-1}/p_{t-1}+ SC_{t-2}/p_{t-2})/3] + E^{o}*[(n_t*q_t/p_t+n_{t-1}*q_{t-1}/p_{t-1}+n_{t-2}*q_{t-2}/p_{t-2})/3] + F^{o}*PE_t + G^{o}*R_t + I^{o}*w^{o}_{t-3} + \varepsilon^{o}_{t}$$

- combined modelling of indoor and outdoor water demands

$$w_t = A + B*[(n_t+n_{t-1}+n_{t-2})/3] + C*[(Y_t/p_t+Y_{t-1}/p_{t-1}+Y_{t-2}/p_{t-2})/3] + Do*[(SC_t/p_t+SC_{t-1}/p_{t-1}+ SC_{t-2}/p_{t-2})/3] + E*[(n_t*q_t/p_t+n_{t-1}*q_{t-1}/n_{t-1}+n_{t-2}*q_{t-2}/p_{t-2})/3] + F*PE_t + G*R_t + H*w_{t-1} + I*w_{t-3} + \varepsilon_t$$

For notation see below. Superscripts i and o refer to indoor and outdoor water use respectively. Subscripts t, t-1, etc. refer to the current and previous periods respectively.

wit = indoor water consumption per house
wot = outdoor water consumption per house
wt = indoor-plus-outdoor water consumption per house
n = household size (number of people per house)
Y = nominal household disposable income
SC = nominal water service charge
p = nominal unit (marginal) water price
q = deflator (consumer price index)
PE = potential evapotranspiration
R = rainfall
ε = error term
υ = error term

A, B, C, ... , H, and I are parameters to be estimated by Least Squares or Maximun Likelihood methods.

B. The Data

The data used correspond to the four municipalities of Greater Victoria and are extensively described in Gai (1988). Income data for 1985 were reported subsequent to the earlier study and they have nov been incorporated into the data. Additional variables have been considered in the econometric models of this report. The new variables are:

1. Potential evapotranspiration.
2. Distribution network's pressure.
3. Dwelling's age.
4. Price index of wages, fuel and materials.
5. Interest rate.
6. Price index of stock of water using appliances.

Data on the nev variables have been added to the data bank and are listed in Appendix G. Potential evapotranspiration data were provided by the Atmospheric Environment Service of Environment Canada for the Victoria Gonzales Heights station. The same agency also provided evapotranspiration and precipitation data for the Victoria International Airport station.

The municipality of Oak Bay provided network pressure charts recorded at one location in the municipality (municipal hall). Data on the age of dwellings were reported in Gai (1988) but this variable was not incorporated into the econometric models of that report, pending the gathering of network pressure data.

A price index for wages, fuel and materials, and the interest rate were

required as two of several supply-side parameters on which the service charge and water price variables were regressed, at the first stage of a Two-Stage Least Square procedure. This method was used to determine whether a simultaneity problem existed with the service charge and price variables.

A price of the stock of water using appliances, the interest rate, and an allowance for depreciation were used to compute a rental price of capital for the stock of appliances, to be used in the stock adjustment models. The price index of that stock was proxied by the price index of the residential capital stock reported by Statistics Canada. Mortgage rates, reported by the Bank of Canada, were used as the representative interest rate.

C. Aggregation Level

As in Gai (1988), municipal average data (municipal totals divided by number of dwellings) were used in the econometric models. The restrictive implications of this simplifying assumption were discussed in Chapter IV.

D. Unit Time Period

For this study, a 4-month time period was adopted for the econometric models. This unit time period corresponds to the billing period in effect in Greater Victoria. In addition, a 4-month period approximately matches the seasonal variation of residential water consumption (winter/summer, indoor/outdoor uses). The calendar year was divided into three periods: January-April, May-August, and September-December.

E. Actual Versus Recorded Water Use

As mentioned in Chapter IV, separate meter readings for indoor and outdoor water uses, leakage, and over or under recording are not available. Therefore, assumptions about meter malfunctioning and leakage were made, prior to modelling the demand for indoor and outdoor water uses. Meter readings were assumed to be unbiased. Leakage water use was assumed to be a linear function of dwelling's age and network's pressure as in Equation (1):

$$v_1 = l_o + l_1 * G + l_2 * H \qquad (1)$$

where:

v_1 = leakage water use per dwelling

G = dwelling's age

H = network's water pressure

l_o, l_1, l_2 = parameters to be estimated

The following assumptions were made in order to estimate leakage losses:

1. Leakage losses are equal to 5 percent of total water consumption for average values of dwelling's age and network's water pressure.
2. Network pressure behaves in a similar way in the four municipalities, and is represented by Oak Bay data. This assumption is required since pressure data for the municipalities of Victoria, Esquimalt and Saanich have not yet been obtained from the records of the respective utilities.

The procedure used to estimate leakage is only a rough approximation, given the available data. The econometric results do improve when leakage losses, as computed by the above procedure, are substracted from total water consumption.

A visual inspection of the Oak Bay pressure charts shows that pressure remains fairly constant on winter days but that it varies considerably on summer days as a result of the increased water consumption for lawn watering. A simultaneity problem, as discussed in Chapter IV, occurs since water consumption influences pressure and pressure in turn influences leakage as well as indoor and outdoor uses. Because of the lack of measured data on leakage, this pressure induced simultaneity problem has been ignored at this time.

F. Indoor and Outdoor Water Uses

Indoor uses occur year round in apartments and houses. Outdoor uses start in March, April or May and stop in September, October or November, depending on the weather conditions of each given year. Therefore, outdoor uses may or may not occur in the January-April and September-December periods, but they always occur in May-August. Since water for indoor and outdoor uses represents two different economic goods, house dwellers may allocate their budget into two or three goods (indoor water use, outdoor water use, and a composite good), depending on weather conditions. Apartment dwellers are assumed to only demand water for indoor use.

In order to model indoor and outdoor water demands separately, total per dwelling water consumption, net of leakage, must be broken down into its indoor and outdoor components. Since indoor and outdoor water consumption are not measured separately, the breakdown can be done based on winter consumption patterns, whereby indoor use in summer is assumed to be equal to or slightly higher than winter's indoor use. Each component can then be modelled separately as a function of the explanatory variables (household size and income, prices, weather variables). In an alternative procedure, the sum of the indoor and outdoor components can be modelled together, but estimates of each component must then be obtained from the estimated indoor-plus-outdoor water consumption according to a given breakdown rule.

Those periods in which outdoor uses do not occur must be excluded from the sample when modelling outdoor water demands, thus reducing the number of observations available. Most of the excluded periods correspond to the January-April period of each year. In contrast, combined modelling allows for the inclusion of all the periods in the sample, since the indoor component occurs in all periods, helping to obtain more reliable estimates.

A full sample is particularly useful in the case of dynamic models, since they include lagged values of the dependent and independent variables, and it is important to track down the adjustments that take place in consecutive periods. The incomplete sample of the outdoor demand case does not completely prevent the tracing of lagged effects. Given the seasonal variations in water consumption, outdoor consumption in the current period is more likely to depend on outdoor consumption in the same period one year

(three periods) before than on consumption in the previous period. However, influences from the previous or second previous periods might still exist because of other factors, such as gradual adjustments in the stock of water-using appliances. Several lag structures can be tried in the econometric models to determine the length of the lag.

Equations for separate and combined modelling of indoor and outdoor water demands are shown for each model in Table 2.

G. Price Variables

As discussed in Chapter IV, the water price variable must be correctly specified when service charges and block rate structures apply. In addition, a potential simultaneity problem associated with block rate structures must be investigated. The water charges that apply in the municipalities of Greater Victoria include a service charge and a uniform price consumption charge, as described in Gai (1988). Under those circumstances, the "difference" variable introduced in chapter IV is equal to the service charge and the marginal price variable is equal to the uniform unit water charge (see Figures 1 and 2). The marginal price and "difference" variables computed for residential customers in Greater Victoria have uniform values, regardless of the amount of water consumed. One of the sources of simultaneity problems is thus eliminated.

H. Estimated Residential Water Demand Functions

Residential water demand functions for indoor-plus-outdoor, indoor, and outdoor uses were estimated using the linear, log-log and comprehensive

models in their static and dynamic (flow-adjustment type) versions. A pooled sample of ninety-nine, 4-monthly observations was used for estimation. This corresponds to eleven years (1976-1986), times three periods per year of data for the municipalities of Victoria/Esquimalt, Oak Bay, and Saanich. Dummy variables not shown on Table 2 were added to identify the municipality corresponding to each observation. A dummy variable that identifies those periods in which outdoor uses occur was added in the indoor-use models.

An allowance for leakage loss was substracted from water consumption per dwelling per period to compute indoor-plus-outdoor water consumption. Leakage loss was estimated from Equation (2):

$$v_1 = 25. + 0.1*G + 0.375*H \quad \text{cubic feet/dwelling/period} \qquad (2)$$

where:

v_1 = leakage loss.

G = average age of dwellings in the municipality.

H = average water pressure in municipal network.

Equation (2) gives a leakage loss of about 5 percent of the recorded consumption for the average dwelling. As mentioned before, econometric results improved when the leakage allowance was substracted from total water consumption.

Indoor and outdoor water use components were computed by assuming that

indoor water use in summer is 10 percent higher than in winter. This assumption was found to improve the estimates. Combined indoor-plus-outdoor and separate indoor and outdoor demands were subsequently modelled. The combined modelling created a problem since potential evapotranspiration and rainfall are irrelevant in winter periods. The problem was handled by defining a deficit variable, as shown in Equation (3):

$$DEFICIT = PE - 0.125*R - 1.85 \quad \text{inches/month} \tag{3}$$

where:

DEFICIT = deficit variable.

PE = potential evapotranspiration.

R = rainfall.

The coefficients in Equation (3) were obtained by regressing the occurrence of outdoor uses (1 for occurrence, 0 for non occurrence) against PE and R. When the deficit is negative, as in winter periods, PE and R are left out of the model by making PE = R = 0.

The six models were fitted to the available data using the Generalized Least Squares (GLS) method in a version proposed by Kmenta (1971) for pooled cross-section time-series data. In this procedure, the Ordinary Least Squares (OLS) is applied first to the data to compute autocorrelation coefficients for each cross section. The data are then corrected for autocorrelation and the OLS method is applied again to the transformed data to compute the variances for each cross section data. The transformed data

are corrected nov for heteroscedasticity and the OLS method is applied one more time to the twice corrected data to obtain the final estimates.

Table 3 shovs the estimated models of indoor-plus-outdoor, indoor, and outdoor vater demands. The dynamic models are of the flov-adjustment type. The table shovs the estimates and statistical significance of the individual coefficients as vell as the fit and significance of each overall model.

Price and income elasticities obtained from each model are listed in Table 4. Short and long run elasticities are shovn for the dynamic models. Short and long run elasticities are identical in the static models, since vater consumption adjustments to explanatory variables changes are assumed to be instantaneous in those models.

Table 5 lists the residential vater demand functions derived from the regressions of Table 3. Figures 3, 4 and 5 shov actual and predicted values of indoor-plus-outdoor vater consumption per house in Victoria/Esquimalt, Oak Bay and Saanich respectively between 1977 and 1986. Two sets of predicted values are compared: those obtained by the combined estimation of indoor-plus-outdoor demand versus those resulting from the addition of the estimates of each component. The estimates shovn correspond to the linear dynamic model.

Table 3. Regression Statistics.
Indoor-plus-Outdoor Water Consumption per House.

MODEL	1.S	1.D	2.S	2.D	3.S	3.D
	LINEAR		LOG-LOG		COMPREHENSIVE	
.	static	dynamic	static	dynamic	static	dynamic .
DEPENDENT VARIABLE	PHT	PHT	PHT	PHT	PHT	PHT .
INDEPENDENT VARIABLES						
NPEOP	361.96	263.75	1.5312	0.64524	355.19	237.32
	(2.21)	(1.50)	(3.93)	(1.54)	(2.23)	(1.78)
RDINC	0.034154	.027612	0.14713	.18276	-	-
	(1.68)	(1.46)	(0.94)	(1.51)		
RSC	4.6484	5.2288	0.10053	.070542	-	-
	(0.41)	(0.51)	(0.84)	(0.59)		
RMP	-67834.	-69126.	-.48260	-.31035	-	-
	(0.77)	(0.85)	(2.10)	(1.84)		
GIPR	-	-	-	-	.8940E-04	.8062E-04
					(1.60)	(1.56)
SCPR	-	-	-	-	.0096710	.011130
					(0.37)	(0.50)
NPMR	-	-	-	-	-.27662	-.16988
					(0.55)	(0.38)
PE	283.18	110.52	0.57151	0.14559	281.23	104.89
	(8.08)	(2.94)	(5.54)	(1.65)	(7.97)	(2.82)
RAIN	-262.26	-120.97	-.32762	-.15819	-249.57	-117.96
	(4.41)	(2.12)	(3.33)	(2.12)	(4.17)	(2.09)
LAGPHT	-	-.079322	-	-.010929	-	-.067848
		(0.89)		(0.14)		(0.75)
LAGPHT3	-	0.56985	-	0.71543	-	0.58795
		(5.56)		(7.93)		(5.70)
CONSTANT	-419.29	-335.32	0.61537	-2.2798	-349.06	-348.36
	(0.81)	(0.70)	(0.24)	(1.18)	(0.84)	(0.98)
BUSE R2	0.5822	0.7488	0.5285	0.8007	0.5759	0.7491
SEE	0.87515	0.86325	0.87310	0.87699	0.86970	0.86229
T-df	83	81	83	81	83	81
F-ndf	6	8	6	8	6	8
F-ddf	83	81	83	81	83	81
F-statistic	19.3	30.2	15.5	40.7	18.8	30.2

Continued.

Table 3. Continued.
Indoor Water Consumption per House.

MODEL	1.S	1.D	2.S	2.D	3.S	3.D
	LINEAR		LOG-LOG		COMPREHENSIVE	
.	static	dynamic	static	dynamic	static	dynamic .
DEPENDENT VARIABLE	PHI	PHI	PHI	PHI	PHI	PHI .
INDEPENDENT VARIABLES						
NPEOP	321.11	272.85	0.72327	0.72911	220.04	152.28
	(2.52)	(3.05)	(1.47)	(2.47)	(2.05)	(1.97)
RDINC	.0050109	.0018924	.052118	-.052206	-	-
	(0.43)	(0.28)	(0.31)	(0.68)		
RSC	5.5984	9.6977	.024706	.085225	-	-
	(0.99)	(1.95)	(0.27)	(1.13)		
RMP	-51135.	-42537.	-.12728	-.16754	-	-
	(1.18)	(1.94)	(0.62)	(1.52)		
GIPR	-	-	-	-	.29553E-4	-.16165E-5
					(0.75)	(0.08)
SCPR	-	-	-	-	.0089861	0.011267
					(0.69)	(1.07)
NPMR	-	-	-	-	-.11340	0.042135
					(0.46)	(0.25)
PET	-	-	-	-	-	-
RAIN	-	-	-	-	-	-
LAGPHI	-	0.57955	-	0.67126	-	0.63373
		(6.08)		(7.84)		(6.78)
DIO	25.180	37.457	.045632	.066116	26.266	38.739
	(2.24)	(2.64)	(2.64)	(2.85)	(2.45)	(2.71)
CONSTANT	-51.102	-398.83	4.6128	0.75553	124.68	-223.29
	(0.17)	(1.71)	(2.08)	(0.63)	(0.40)	(1.19)
BUSE R2	0.1761	0.6720	0.1542	0.6808	0.1602	0.6163
SEE	0.90577	0.88500	0.91810	0.90050	0.90775	0.88678
T-df	84	83	84	83	84	83
F-ndf	5	6	5	6	5	6
F-ddf	84	83	84	83	84	83
F-statistic	3.59	28.3	3.06	29.5	3.20	22.2

Continued.

Table 3. Continued.
Outdoor Water Consumption per House.

MODEL	1.S	1.D	2.S	2.D	3.S	3.D
	LINEAR		LOG-LOG		COMPREHENSIVE	
.	static	dynamic	static	dynamic	static	dynamic .
DEPENDENT VARIABLE	PHO	PHO	PHO	PHO	PHO	PHO .
INDEPENDENT VARIABLES						
NPEOP	-499.05	-376.38	1.0466	3.1286	72.966	4.7256
	(1.40)	(0.97)	(0.42)	(0.97)	(0.26)	(0.02)
RDINC	.035789	.015273	**1.9972**	**2.2228**	-	-
	(1.02)	(0.37)	**(2.07)**	**(2.25)**		
RSC	-4.1935	-16.058	-.0045599	0.26874	-	-
	(0.24)	(0.77)	(0.01)	(0.30)		
RMP	288750.	174640.	0.19877	-.65540	-	-
	(1.55)	(0.89)	(0.12)	(0.36)		
GIPR	-	-	-	-	.00010264	.59637E-4
					(0.83)	(0.42)
SCPR	-	-	-	-	.0042309	-.025058
					(0.10)	(0.48)
NPMR	-	-	-	-	-1.1432	-0.27281
					(1.21)	(0.24)
PET	**471.99**	270.29	**6.2103**	**4.7475**	**471.37**	273.11
	(4.03)	(1.62)	**(5.36)**	**(3.01)**	**(4.27)**	(1.61)
RAIN	-51.435	-182.44	0.14046	-.21613	-87.070	-211.64
	(0.34)	(1.07)	(0.26)	(0.36)	(0.62)	(1.26)
LAGPH03	-	0.28405	-	0.14172	-	0.27972
		(1.60)		(1.46)		(1.49)
CONSTANT	-763.86	235.87	-19.382	-27.871	-354.77	-24.725
	(0.74)	(0.19)	(1.07)	(1.47)	(0.34)	(0.02)
BUSE R2	0.7252	0.7553	0.7504	0.8134	0.7321	0.7455
SEE	0.78322	0.74359	0.77300	0.73756	0.78192	0.74145
T-df	38	37	38	37	38	37
F-ndf	6	7	6	7	6	7
F-ddf	38	37	38	37	38	37
F-statistic	**16.7**	**16.3**	**19.0**	**23.0**	**17.3**	**15.5**

Continued.

Table 3. Concluded.

NOTES: For definition of variables see below.
All variables in the log-log model are in logarithmic form, except DIO and CONSTANT.
Independent variables in dynamic models are averages of current and two previous periods, with the following exceptions: PE and RAIN refer to the current period only; the lagged dependent variables correspond to the previous period (LAGPHT, LAGPHI), and the period one year before (LAGPHT3, LAGPHO3).
T-statistics are in parentheses.
BUSE R2 = Buse's R-square (Buse, 1973).
SEE = standard error of the estimate.
T-df = degrees of freedom in T-statistic.
F-ndf = numerator's degrees of freedom in F-statistic.
F-ddf = denominator's degrees of freedom in F-statistic.

PHT = indoor-plus-outdoor water consumption in cubic feet per house per month.
PHI = indoor water consumption in cubic feet per house per month.
PHO = outdoor water consumption in cubic feet per house per month.
NPEOP = household size (number of people per house).
RDINC = real household disposable income in 1981 $/year.
RSC = real service charge in 1981 $/4-month period.
RMP = real unit (marginal) price of water in 1981 $/100 cubic feet.
GIPR = (household income / water price) ratio.
SCPR = (service charge / water price) ratio.
NPMR = (deflator / water price) ratio times household size.
PE = potential evapotranspiration in inches per month.
RAIN = rainfall in inches per month.
LAGPHT = previous period's indoor-plus-outdoor water consumption in cubic feet per house per month.
LAGPHT3 = indoor-plus-outdoor water consumption in the same period one year before, in cubic feet per house per month.
LAGPHI = previous period's indoor water consumption in cubic feet per house per month.
LAGPHO3 = outdoor water consumption in the same period one year before, in cubic feet per house per month.
DIO = dummy variable for periods in which outdoor consumption occurs.
CONSTANT = intercept of the regression line.

Table 4. Price and Income Elasticities of Demand.

MODELLING	MODEL	PRICE ELASTICITY		INCOME ELASTICITY	
		Short Run	Long Run	Short Run	Long Run
indoor plus outdoor	1.S	-0.14	-0.14	0.19	0.19
	1.D	-0.05	-0.29	0.06	0.32
	2.S	-0.48	-0.48	0.15	0.15
	2.D	-0.10	-1.05	0.06	0.62
	3.S	-0.04	-0.04	0.18	0.18
	3.D	-0.03	-0.20	0.06	0.35
indoor	1.S	-0.22	-0.22	0.06	0.06
	1.D	-0.06	-0.43	0.01	0.05
	2.S	-0.13	-0.13	0.05	0.05
	2.D	-0.06	-0.51	-0.02	-0.16
	3.S	-0.04	-0.04	0.12	0.12
	3.D	-0.04	-0.31	-0.002	-0.02
outdoor	1.S	0.89	0.89	0.30	0.30
	1.D	0.18	0.74	0.04	0.18
	2.S	0.20	0.20	2.00	2.00
	2.D	-0.22	-0.76	0.74	2.59
	3.S	0.68	0.68	0.30	0.30
	3.D	0.04	0.18	0.05	0.19

Elasticities in the linear models 1.S and 1.D are computed at the mean values of the independent variables. Elasticities in the log-log models 2.S and 2.D are constant throughout the whole range of the independent variables. Short and long run elasticities are identical by definition in the static models 1.S, 2.S and 3.S .

Table 5. Residential Water Demand Functions.

1.S Linear Static Model

$$PHT_t = -419.29 + 361.96*NPEOP_t + 0.034154*RDINC_t + 4.6484*RSC_t - 67834.*RMP_t + 283.18*PE_t - 262.26*RAIN_t$$

1.D Linear Dynamic Model

$$PHT_t = -335.32 + 263.75*[(NPEOP_t+NPEOP_{t-1}+NPEOP_{t-2})/3] + 0.027612*[(RDINC_t+RDINC_{t-1}+RDINC_{t-2})/3] + 5.2288*[(RSC_t+RSC_{t-1}+RSC_{t-2})/3] - 69126.*[(RMP_t+RMP_{t-1}+RMP_{t-2})/3] + 110.52*PE_t - 120.97*RAIN_t - 0.079322*PHT_{t-1} + 0.56985*PHT_{t-3}$$

2.S Log-log Static Model

$$PHT_t = 1.8503*NPEOP_t^{1.5312} RDINC_t^{0.14713} RSC_t^{0.10053} RMP_t^{-0.48260} PE_t^{0.57151} RAIN_t^{-0.32762}$$

2.D Log-log Dynamic Model

$$PHT_t = 0.10230*[(NPEOP_t NPEOP_{t-1} NPEOP_{t-2})^{1/3}]^{1.1179} [(RDINC_t RDINC_{t-1} RDINC_{t-2})^{1/3}]^{0.14713} [(RSC_t RSC_{t-1} RSC_{t-2})^{1/3}]^{0.070542} [(RMP_t RMP_{t-1} RMP_{t-2})^{1/3}]^{-0.31035} PE_t^{0.14559} RAIN_t^{-0.15819} PHT_{t-1}^{-0.010929} PHT_{t-3}^{0.71543}$$

Continued.

Table 5. Concluded.

3.S Comprehensive Static Model

$$PHT_t = -349.06 + 355.19*NPEOP_t + 0.000089398*GIPR_t + 0.0096710*SCPR_t - 0.27662*NPMR_t + 281.23*PE_t - 249.57*RAIN_t$$

3.D Comprehensive Dynamic Model

$$PHT_t = -348.36 + 237.32*[(NPEOP_t+NPEOP_{t-1}+NPEOP_{t-2})/3] + 0.000080624*[(GIPR_t+GIPR_{t-1}+GIPR_{t-2})/3] + 0.011130*[(SCPR_t+SCPR_{t-1}+SCPR_{t-2})/3] - 0.16988*[(NPMR_t+NPMR_{t-1}+NPMR_{t-2})/3] + 104.89*PE_t - 117.96*RAIN_t - 0.067848*PHT_{t-1} + 0.58795*PHT_{t-3}$$

NOTE: For definition of variables see Table 3.

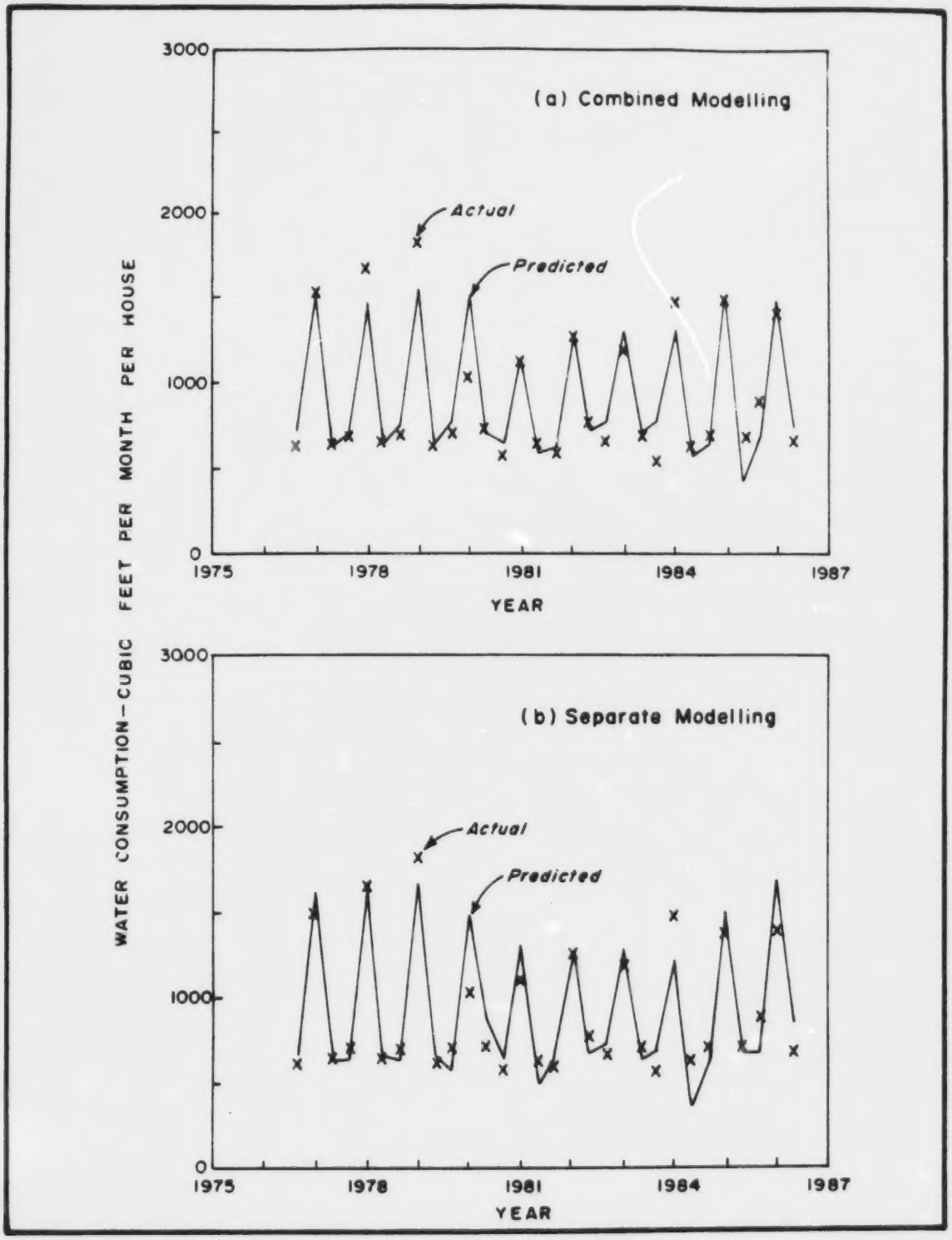

Figure 3 Victoria / Esquimalt : Actual and predicted water consumption per house.

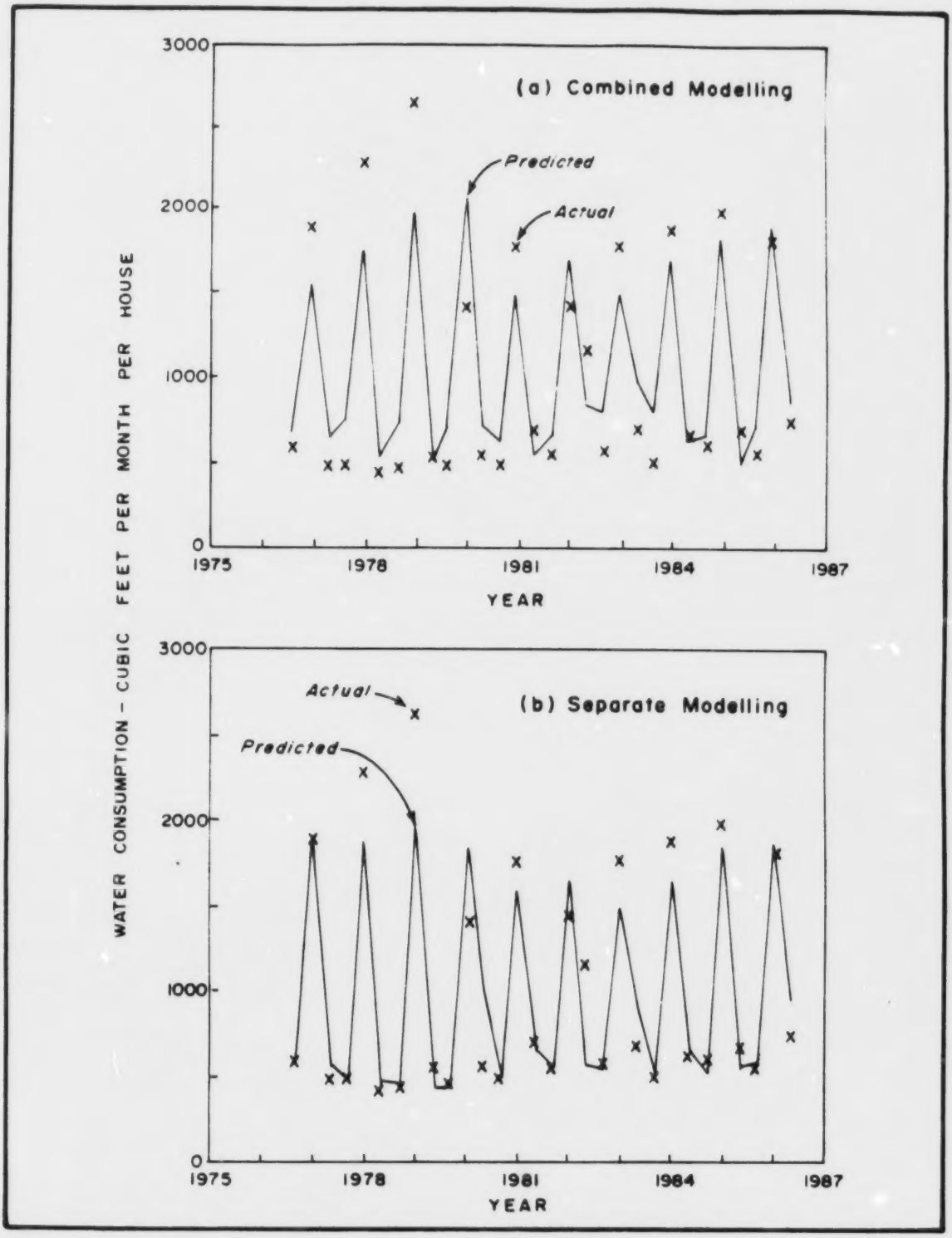

Figure 4 Oak Bay : Actual and predicted water consumption per house.

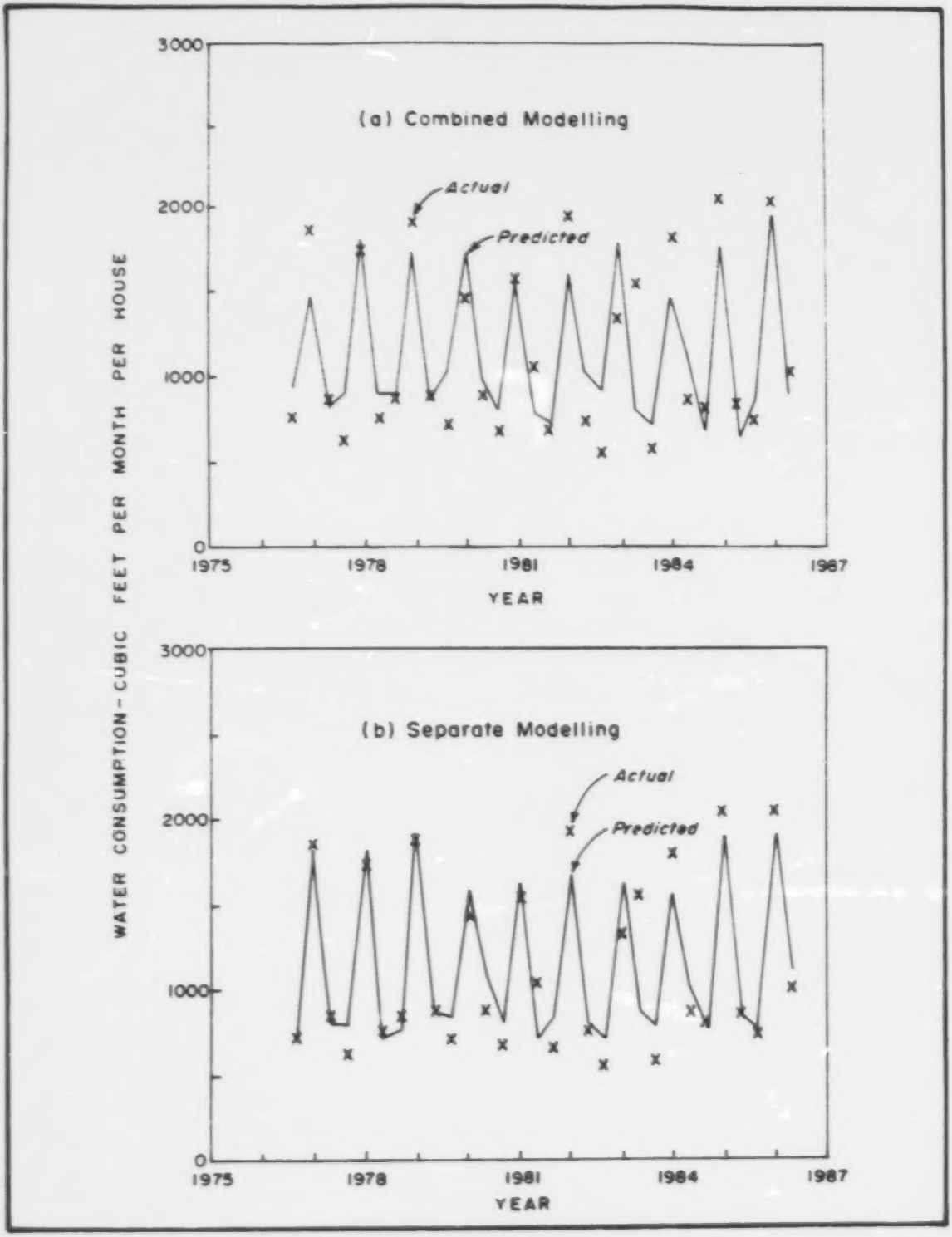

Figure 5 Saanich : Actual and predicted water consumption per house.

I. Discussion of Econometric Results

The results of the combined modelling of indoor-plus-outdoor demands are better than those obtained by modelling each component separately; however, the latter procedure is required to obtain separate elasticities of indoor and outdoor demands. Separate modelling highlighted the different influence of a given variable on each component demand. For instance, household size has a greater influence on indoor than on outdoor water demands and the opposite is true for price and income, while potential evapotranspiration and rainfall precipitation do not influence indoor demands, at least in winter.[3]

As mentioned before, separate recording of indoor and outdoor water consumption was not available in the study area. Therefore, an assumption was made about the relative shares of each type of use when modelling them separately. Indoor water uses in summer were estimated based on winter consumption patterns, since most water uses in winter are of the indoor type. A similar assumption has been used in other studies such as Howe and Linaweaver (1967). It can be argued that separate modelling conducted under an alternative breakdown assumption might produce better estimates than combined modelling. The issue could be settled if separate recording of indoor and outdoor water consumption were conducted.

The dynamic models performed better than the static ones, particularly in

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3. The latter two variables indirectly influence indoor demands in summer by affecting outdoor demands and forcing a reallocation of the household's budget between indoor and outdoor water use and all other goods and services.

the case of indoor water demands. The improvement was marginal in the case of outdoor demands, and intermediate in the combined case. These results indicate that indoor water demand tends to follow historical trends whereas outdoor consumption responds mainly to current weather conditions rather than to the trend of consumption in previous years. A gradual adjustment process takes place, therefore, at least for indoor water consumption.

The municipal dummy variables were found not to be significant and were left out of the model. The different rates of water consumption across municipalities seem to be fully explained by differences in household income and size, and water charges. The dummy variable that identifies the periods of outdoor uses is significant in the indoor use models, indicating that indoor water consumption in summer is affected by the existence of outdoor water uses.

The fit of the models is low for indoor demands, high for outdoor demands and moderately high for combined demands. The statistical significance of each model is high, as measured by the F-statistic, except for the static models of indoor demand which are marginally significant. In general, the individual coefficients show the right sign in indoor-plus-outdoor and indoor modelling although they are not always significant. Combined modelling is therefore more reliable.

The presence of a lagged dependent variable as one of the explanatory variables in the dynamic models creates an econometric problem and produces biased estimates. The severity of this problem was studied by conducting a

Monte Carlo simulation and computing the bias of the GLS estimators. The magnitude of the bias was found to range between ten and fifty percent, depending on the variable. Special econometric methods would have to be used to produce unbiased estimates.

The coefficients of the service charge consistently show the wrong sign although the estimates are not significant. A potential simultaneity problem in the service charge could have accounted for its wrong sign. Two-Stage Least Squares were used to test for this possibility. The service charge and the price variables were expressed as linear functions of supply-side variables such as number of dwellings served, price index of wages, fuel and materials, interest rate, time, and municipal dummy variables. The price indexes corresponded to industrial sectors or the whole economy, as cost data have not been gathered from the municipal utilities of Greater Victoria. The service charge and water price equations, together with the demand equation, formed a system of equations and the demand equation was reestimated using Two-Stage Least Squares. The Two-Stage Least Squares estimates, shown in Appendix I, also have the wrong sign on the service charge. Two-Stage estimation that used cost data specific to the utilities of the study area might produce a service charge coefficient with the right sign.

The coefficient of the previous period's consumption in the combined models shows a wrong sign and low significance. The same is true of the coefficients of the economic variables in the comprehensive model, indoor-use case. When outdoor water uses are modelled separately, the main

unexpected results are a wrong sign and low significance of the price coefficient, and a right sign but low significance of the rain coefficient. The Two-Stage Least Squares estimation did not correct the sign of the price coefficient, but this result might change if Greater Victoria cost data were available.

The price and income elasticities obtained from the combined and separate modelling described above show the right sign, except for the price elasticities of outdoor demands. The magnitude of the elasticities are similar to those obtained in other studies. The price and income elasticities are small for indoor demands and the income elasticity of indoor demand is smaller than the outdoor elasticity, since indoor water uses satisfy more basic needs. Long run elasticities are much larger than short run elasticities, indicating that water consumption adjusts over several billing periods in response to changes in the explanatory variables.

Appendix J presents a comparison of the estimates obtained using comprehensive models of the stock-adjustment and flow-adjustment types. Both models are derived in Appendix G. Each explanatory variable of the comprehensive flow-adjustment model is the average of the current and two previous periods. In the stock-adjustment model each variable is represented by its current, lagged, and average value of the three previous periods. In addition, each of these three components is pre-multiplied by different ratios of the prices of water, other goods and stock of apliances. Furthermore, the current and lagged variable terms always have

the opposite sign in this model. Given the high correlation between lagged values of most explanatory variables, multicollinearity problems exist and the estimates are rather poor. Only the current weather variables show the right sign and are statistically significant. The lagged dependent variable shows a negative sign, implying a very large stock adjustment coefficient and short-run elasticities that exceed the long-run elasticities.

In conclusion, the econometric results indicate that combined modelling, using the comprehensive dynamic model, should be used at this stage for forecasting purposes. The coefficients of the comprehensive dynamic model under combined modelling show the right sign, except the service charge, and the model produces short and long run price and income elasticities of reasonable magnitude. Two-Stage estimates using cost data, if obtained from the water utilities of Greater Victoria, could improve the results by producing a service charge coefficient with the right sign. Although the performance of the linear dynamic model is similar to that of the comprehensive model, the latter produces better forecasts outside of the range of the historical data.

J. Summary and Conclusions

The results of econometric work carried out using six econometric models for combined and separate modelling of indoor and outdoor household water demands in Greater Victoria indicate the following:

1. Combined modelling of indoor plus outdoor water demands produced better econometric results than separate modelling. On the other

hand, separate modelling allowed for the identification of the variables that explain each component and the estimation of separate indoor and outdoor price and income elasticities.

2. Indoor demand is determined mainly by household size and previous consumption.
3. Outdoor demand depends mainly on the weather variables. Income and previous consumption have some influence.
4. The estimated short and long run price and income elasticities of demand are of a reasonable magnitude and, in most cases, of the expected sign.
5. The coefficients of the service charge variable, and in some cases marginal price, exhibit wrong signs. This result could be attributed to simultaneity problems present in those two variables. Although the application of Two-Stage Least Squares did not improve the results, the use of cost data specific to the utilities of the study area might improve study results.
6. The performance of the comprehensive model in its static and dynamic versions was very similar to that of the linear model. However, the comprehensive model performs better in extrapolations required for forecasting.
7. The performance of the stock-adjustment version of the comprehensive model was rather poor.

VI. DEMAND FORECASTING AND VALUE OF WATER

This chapter presents two applications of the residential water demand functions derived in the previous chapter:

1. Water demand forecasting.
2. Estimation of the value residential customers attach to the water they use.

Demand forecasting is crucial for the planning of expansions of the water supply system made necessary by demand growth. In the traditional supply-side management approach, the consumers' valuation of additional water supplies is compared with the cost of the incremental supply to determine the need for and timing of expansions. In the demand-side management approach, price is used as a tool to control demand growth and restore the supply/demand balance. In this case, the consumers' water valuation is used to compute the loss in consumer surplus induced by the higher price. This loss is compared to the savings realized from the postponement of the system expansion.

A. Demand Forecasting

Figure 6 illustrates an example of demand forecasting. The comprehensive dynamic model has been used to generate a demand forecast for the Victoria/Esquimalt municipal area for the period 1987-1991 in panel [a]. The forecast is based on the following assumptions:

1. The 1981-1986 and 1986-1991 growth rates of the number of dwellings are the same.

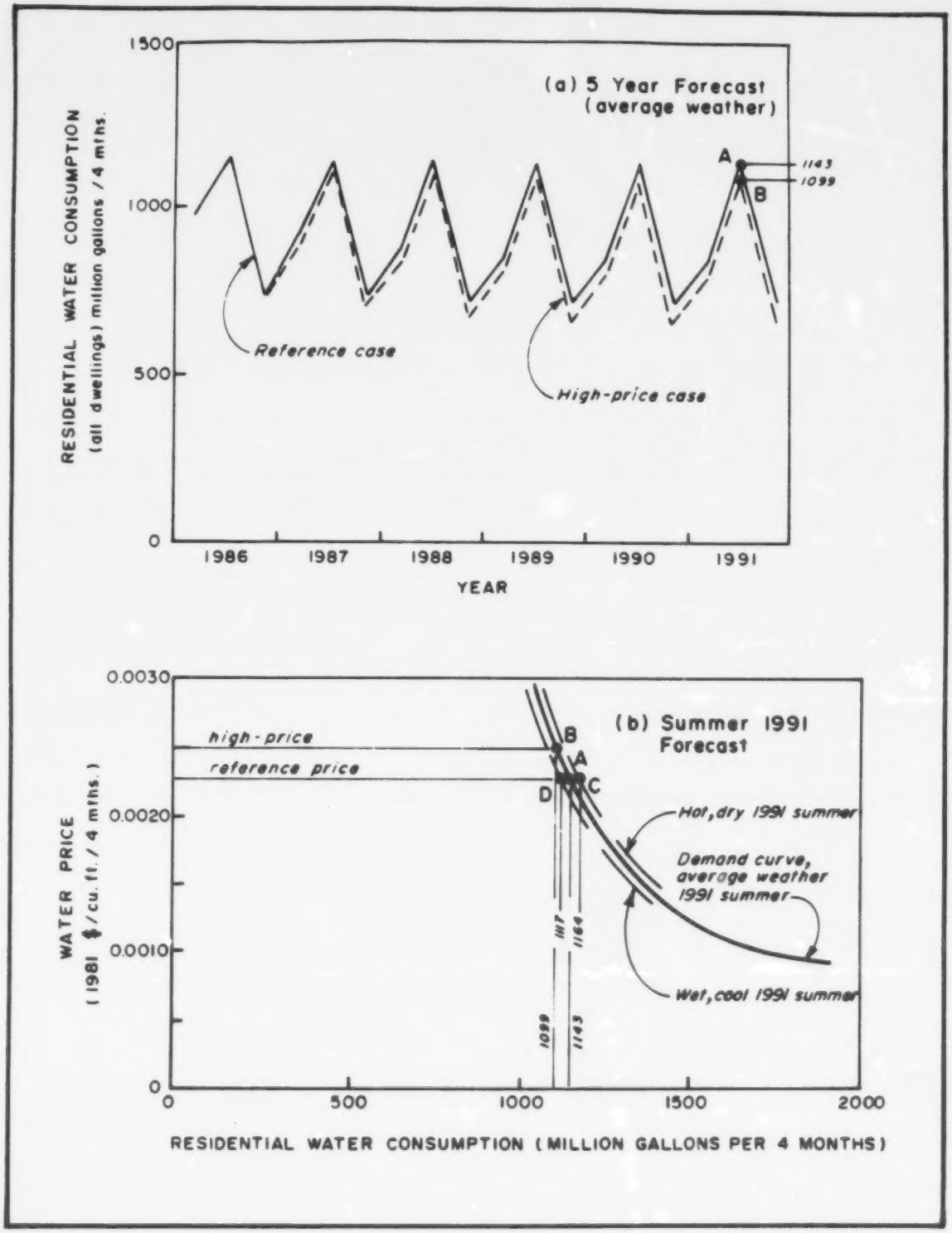

Figure 6 Victoria / Esquimalt : Water Demand Forecast

2. Household size, real household income and water service charge remain at their 1986 values.
3. Real marginal price remains at its 1986 value. This is called the reference forecast. A second forecast, the high-price forecast, is prepared for an assumed 10 percent real marginal price increase imposed in January 1987.
4. Average weather occurs in every 4-month period of the 1987-1991 time period.

As expected, residential water demand in the reference forecast is slightly larger than in the high-price forecast. Points A and B in panel [a] correspond to points A and B in panel [b]. The latter panel shows the demand curve for the summer of 1991. The assumed 10 percent real price increase of January 1987 produces a 3.8 percent decrease in consumption in the summer of 1991 (price elasticity = -0.38). A comparable variation in consumption would occur if weather conditions that summer were to deviate from the average. As points C and D in panel [b] show, a hot dry summer and a wet cool one would produce a 1.8 percent increase and a 2.3 percent decrease in water consumption respectively. The impact of weather on consumption is mitigated by the fact that about 60 percent of the dwellings are apartments which the econometric model assumes do not consume water for outdoor uses. The weather impact would be much higher in the municipalities of Oak Bay and Saanich, where houses make up for more than 80 percent of the dwellings.

B. Value of Water

The demand curve of panel [b] in Figure 6 corresponds to the water consumption for indoor and outdoor uses and leakage of all the dwellings in the municipality. This demand curve is relevant from the utility's point of view since leakage consumption represents another demand to be met. However, residential leakage paid for by residential customers does not contribute to their utility. Per-dwelling demand curves shown in Figure 7 are used to compute the value consumers attach to water and their consumer surplus. These demand curves identify the three water uses - indoor, outdoor, and leakage. By assumption, there is no outdoor consumption in apartments. In addition, indoor water demand is assumed to be the same in apartments and houses, given equal levels of other explanatory variables such as household size and income.

Panel [a] in Figure 7 shows the forecast of water demand per apartment in the Victoria/Esquimalt municipal area in the summer of 1991. The indoor plus outdoor consumption at a price of 0.0023 $(1981)/cubic-foot/4-months would be equal to 3036 cubic-feet/4-months. A leakage allowance of 220 cubic-feet/4-months must be added. The value of water to a family living in an apartment is at least equal to the area OCC'BD, since the graph has been cut off at the 0.0150 $(1981)/cubic-foot/4-months price. The water charges are made up of a consumption charge equal to the price of water times the sum of the useful consumption and the leakage (areas OABD and DBB'D'), and a service charge of 19 $(1981)/4-months. For the sake of geometric exposition, the service charge is shown as an additional consumption charge. The consumer surplus per apartment is equal to

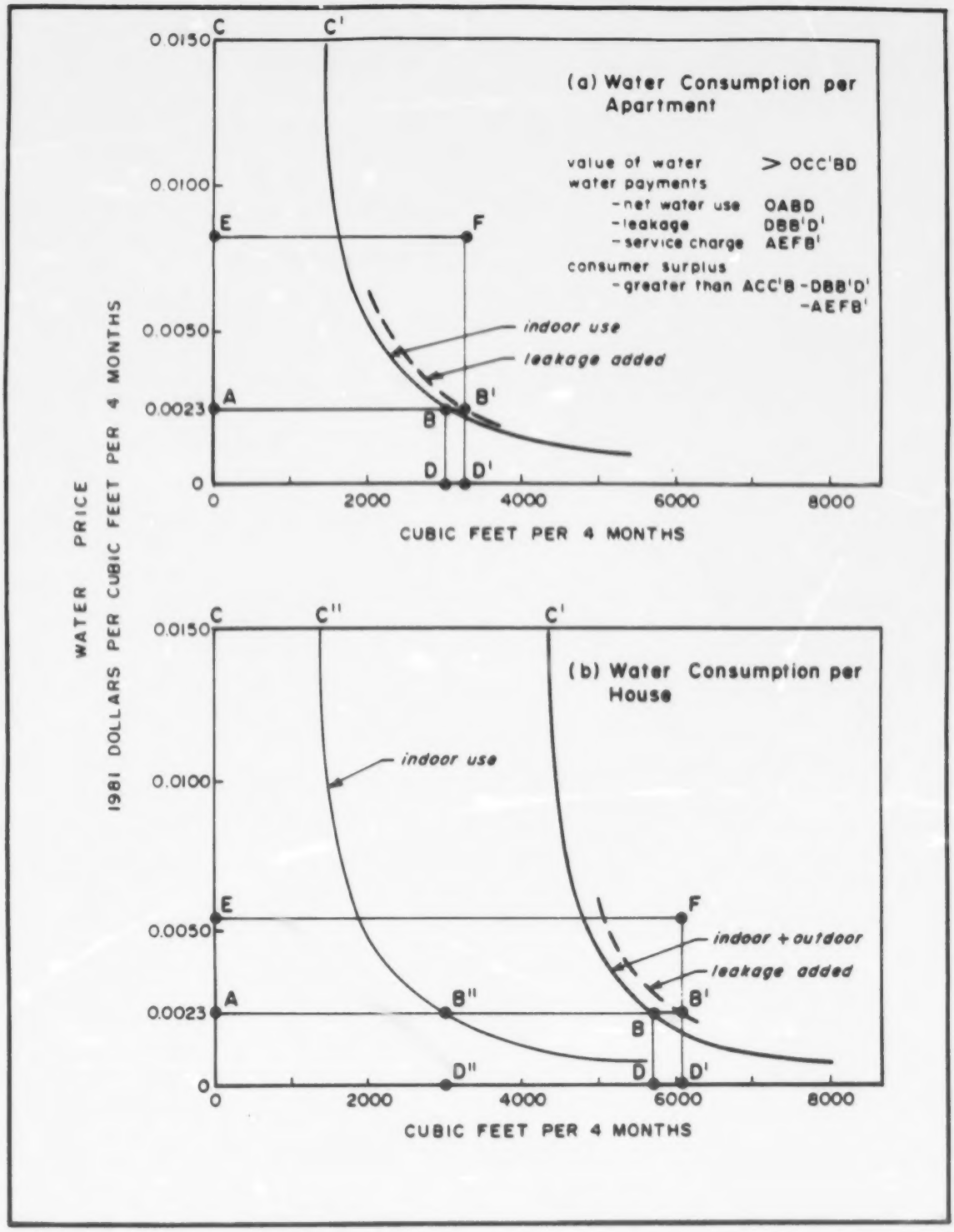

Figure 7 Victoria / Esquimalt : Value of Water (Summer 1991)

4.30 $(1981)/4-months up to the 0.0150 $(1981)/cubic-foot/4-months price. A similar exercise is conducted in panel [b] for a house. Outdoor uses occur in this case and the consumer surplus per house is equal to 42.0 $(1981)/4-months up to the same cut-off price. The totals obtained by adding across the 25,000 apartments and 17,000 houses in the two municipalities are:

Value of water:	1 641 000	(1981)$/4-months
Service charge:	- 798 000	" "
Indoor+outdoor charge:	- 403 000	" "
Value lost to leakage:	- 21 000	" "
Consumer Surplus:	419 000	(1981)$/4-months

The actual consumer surplus is much higher than the above figure since the area under the demand curve and above the arbitrary cut-off price of 0.0150 $(1981)/cubic-foot/4-month is substantial. As price increases, the water demand curve approaches a subsistence level in an asymptotic way and consumers attach an extremely high value to water consumption up to that level. Consumption beyond the subsistence level is valued less.

Water valuation exercises using a linear demand function present an extrapolation problem. The linear demand function will intersect the price axis implying that water is not demanded above a certain price. This feature of the linear demand is not consistent with actual consumption behaviour, since some amount of water is required for subsistence and will be demanded regardless of price.

VII. CONCLUSIONS

This study has investigated additional methodological approaches identified in Gai (1988). The theoretical and econometric specifications of modelling residential water demands have been examined in greater depth. A linear model and a comprehensive econometric model derived from economic theory have been estimated as an alternative to the log-log model used in Gai (1988). Static and dynamic versions of each model were estimated to allow for the gradual adjustment of residential water consumption in response to changes in the explanatory variables. The statistical performance of flow-adjustment and stock-adjustment dynamic models was also investigated. Simultaneity problems induced by potential supply-side effects embedded in the water charges variables were investigated using Two-Stage Least Squares.

Additional data gathered since the completion of Gai (1988) were incorporated in the estimation. Potential evapotranspiration replaced temperature in all models. An allowance for leakage, based on distribution's network pressure and age of dwellings, was substracted from the dwelling's water billed prior to estimation. A price index of the stock of water-using appliances was incorporated in the stock-adjustment dynamic model.

The overall statistical performance of the models is satisfactory, with some models performing better than others. The estimation results are better for combined indoor-plus-outdoor water demand than for separate

modelling. The statistical performance of the comprehensive and linear models in their static and dynamic versions is similar. The comprehensive model produces better forecasts when extrapolation beyond the range of the historical data is required.

Estimation of dynamic models with pooled cross-section time-series data is complex when there is a lagged dependent variable. The derivation of a special econometric method is required to properly deal with this situation. The results of a Monte Carlo simulation suggest that the estimates reported for the dynamic models are biased enough to warrant the use of special econometric techniques in future studies to provide unbiased estimates.

Potential simultaneity problems introduced by the service charge and price variables were investigated using Two-Stage Least Squares. The two variables were regressed on the number of dwellings, interest rate, price index of wages, fuel and materials, municipal dummy variables and time. The results of the two-stage estimation were poor, but regression of the two variables on cost data provided by the utilities might improve the results. Cost data have not been gathered from the utilities.

The econometric results of this study are, in general, consistent with results obtained in studies conducted in other regions and with the predictions of economic theory. A few unexpected results remain to be explained, particularly the wrong signs of the service charge coefficient in indoor and outdoor uses, and price in the outdoor-use case. It is

likely that the unexpected results are due to a combination of the following factors:

1. Aggregation problems.
2. Small variability of data on price and other explanatory variables.
3. Errors in the construction of the dependent variable.
4. Simultaneity problems in the service charge and price variables.
5. Bias due to the lagged dependent variable in dynamic models.

In conclusion, appropriate methods for the estimation of residential water demand in the study area have been identified and developed in this study and the most relevant explanatory variables have been singled out. Some estimation problems remain to be addressed in the case of dynamic models. Likevise, simultaneity problems could be addressed if cost data were obtained from the relevant utilities. Until additional data are gathered and the remaining econometric problems are solved, residential water demand in the study area should be forecast using the comprehensive dynamic model in its combined-modelling version. A simulation approach might provide an alternative modelling strategy to the econometric approach, but it would share the same data problems.

The results of this study indicate a number of areas where further study could improve the accuracy of the models. The main obstacle for additional progress continues to be the quality and aggregation of some of the data and the lack of data on some explanatory variables that have been proxied by related variables. The use of potential evapotranspiration instead of temperature has improved the results, but additional work should be done to

better model outdoor water demands. For instance, in addition to potential evapotranspiration and rainfall, soil moisture conditions are likely to influence outdoor demands.

The selection of a test area for the collection of disaggregated data on water consumption and explanatory variables through continuous metering and questionnaires respectively, remains the best, albeit expensive, strategy for future studies. Alternatively, the methodology used in Gai (1988) and this subsequent study could be refined by using data gathered for other residential water demand studies such as the Saskatchewan study (Brockman and Kulshreshtha 1988), or data stored in Environment Canada's Municipal Water Use Database (Lacelle 1986).

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APPENDICES

APPENDIX A

A Cobb-Douglas Model of Residential Water Demand

This appendix derives an econometric model of residential water demand based on the economic theory of consumer demand that uses a Cobb-Douglas utility function. The model assumes an economy with only 2 goods, water (w) at price (p) and a composite good (z) at price (q), and one representative consumer unit (household) of size (n) and income (Y). Consumer preferences are represented by a Cobb-Douglas utility function:

$$U(w,z,n,PE,R) = K*n*[(w+R-PE)/n]^{a}(z/n)^{b} \qquad (1)$$

where:

- U = Utility function
- w = water consumption
- z = composite good consumption
- n = household size
- PE = potential evapotranspiration
- R = rainfall
- K,a,b = parameters of the utility function

It is assumed in this model that household utility is equal to the sum of the utilities of its members and that each member consumes the same amounts of water and composite good. Potential evapotranspiration and rainfall are included in the utility function since they tend to respectively increase and decrease the demand for lawn watering.

The consumer's problem is:

Select w,z to maximize U(w,z,n,PE,R)

given K,a,b,n,PE,R,p,q

and subject to $Y = p*w + q*z$

The problem can be solved using Lagrange's method (Varian, 1984). The solution to the problem is:

$$w = (a/(a+b))(Y/p) + (b/(a+b))(PE-R) \qquad (2)$$

$$z = (b/(a+b))(Y/q) - (b/(a+b))(p/q)(PE-R) \qquad (3)$$

The solution does not depend on household size n. Residential water demand in this model is also independent of the price of the composite (all others) good.

Equation (2) can be written as in (4) for estimating purposes:

$$w = A + B*(Y/p) + C*PE + D*R + \varepsilon \qquad (4)$$

where $A = 0$, $B = a/(a+b)$, $C = -D = b/(a+b)$, and ε = error term.

The solution is linear on potential evapotranspiration, rain and the ratio of household income to water price. The model can be estimated using standard econometric methods. The model restricts the constant term (intercept) to be zero and the coefficients of potential evapotranspiration and rainfall to be equal and of opposite sign.

The price and income elasticities of water demand can be computed as:

$$e_w = -(a/(a+b))(Y/p)/[(a/(a+b))(Y/p)+(b/(a+b))(PE-R)] \qquad (5)$$

$$m_w = (a/(a+b))(Y/p)/[(a/(a+b))(Y/p)+(b/(a+b))(PE-R)] \qquad (6)$$

where:

e_w = price elasticity of residential water demand

m_w = income elasticity of residential water demand

APPENDIX B

A Stone-Geary Model of Residential Water Demand

This model is similar to that of Appendix A, but a Stone-Geary function is used instead:

$$U(w,z,n,PE,R,w_o,z_o) = K*n*[(w+R-PE)/n-w_o]^a(z/n-z_o)^b \quad (1)$$

where all the variables are as defined in Appendix A, and w_o and z_o are the per capita subsistence level of water and the composite good respectively. The Stone-Geary function implies that part of the consumer's budget is allocated first to buy the subsistence levels w_o and z_o. The remainder of the budget is then allocated to additional amounts of water and the composite good in such a way as to maximize the consumer's utility.

The consumer's problem is:

Select w,z to maximize $U(w,z,n,PE,R,w_o,z_o)$

given K,a,b,n,PE,R,w_o,z_o,p,q

and subject to $Y = p*w + q*z$

The solution to the problem is:

$$w = (b/(a+b))n*w_o + (a/(a+b))(Y/p) + (b/(a+b))(PE-R) - (a/(a+b))(q/p)n*z_o \quad (2)$$

$$z = -(b/(a+b))(p/q)n*w_o + (b/(a+b))(Y/q) - (b/(a+b))(p/q)(PE-R) + (a/(a+b))n*z_o \quad (3)$$

Equation (2) can be rewritten as in (4) for estimating purposes:

$$w = A + B*n + C*(Y/p) + D*PE + E*R + F*n*(q/p) + \varepsilon \quad (4)$$

where:

$A = 0$
$B = (b/(a+b))*w_o$
$C = a/(a+b)$
$D = b/(a+b)$
$E = -b/(a+b)$
$F = (a/(a+b))*z_o$
ε = error term

The solution is non linear on income Y and prices p and q. However, the model is linear on n, PE, R, and the variables Y/p and n*(q/p). As such, the model can be estimated using standard econometric methods. The subsistence levels can be computed as:

$$w_o = \hat{B}/\hat{D}$$

$$z_o = \hat{F}/\hat{C}$$

where $\hat{B}$, $\hat{C}$, $\hat{D}$ and $\hat{F}$ are the estimates of B, C, D and F.

Estimates of the parameters a and b cannot be retrieved from the estimated coefficients of the model. However, if the usual normalization a + b = 1 is used, then C = a ; D = b = 1-a = 1-C .

The price and income elasticities of water demand can be computed as:

$$e_w = -[C*(Y/p)+F*n*(q/p)] \;/\; [A+B*n+C*(Y/p)+D*PE+E*R+F*n*(p/q)] \quad (5)$$

$$m_w = [C*(Y/p)] \;/\; [A+B*n+C*(Y/p)+D*PE+E*R+F*n*(p/q)] \quad (6)$$

where $\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{D}$, $\hat{E}$ and $\hat{F}$ are the estimates of A, B, C, D, E and F.

APPENDIX C

A CES Model of Residential Water Demand

This model is similar to those of Appendixes A and B but a Constant Elasticity of Substitution (CES) utility function is used:

$$U(w,z,n,PE,R) = n*[((w+R-PE)/n)^{\rho} + (z/n)^{\rho}] \quad (1)$$

where all the variables are as defined in Appendix A, and ρ is a parameter of the utility function. The CES is a fairly general utility function and it can be shown that the Cobb-Douglas function is a particular case of it.

The consumer's problem is:

Select w,z to maximize U(w,z,n,PE,R)

given ρ,n,PE,R

and subject to $Y = p*w + q*z$

The solution to the problem is:

$$w = (p^r/(p^r+q^r))(Y/p) + (q^r/(p^r+q^r))(PE-R) \quad (2)$$

$$z = (q^r/(p^r+q^r))(Y/q) - (q^r/(p^r+q^r))(p/q)(PE-R) \quad (3)$$

where: $r = \rho/(\rho-1)$.

The solution does not depend on household size n. Equation (2) must be estimated using non-linear estimation techniques since the only coefficient to be estimated, r, appears as an exponent of prices throughout.

The price and income elasticities of water demand can be computed as:

$$e_w = (p^r/(p^r+q^r))[(r*p^r+(p^r+q^r)(r-1))(Y/p)+r*q^r*(PE-R))/(p^r(Y/p)+q^r(PE-R))]$$

$$m_w = p^{(r-1)}Y[p^r(Y/p)+q^r(PE-R)]$$

APPENDIX D

DYNAMIC MODELS OF RESIDENTIAL WATER DEMAND

1. Flow Adjustment Model

This model assumes that the actual level of consumption adjusts partially towards a desired level of consumption, as described in equations (1) and (2):

$$w_t = w_{t-1} + \theta(w_t^* - w_{t-1}) = (1-\theta)wt-1 + \theta * w_t^d \quad (1)$$

$$w_t^d = F(n_t, Y_t, p_t, q_t, PE_t, R_t) \quad (2)$$

where:

w_t = actual level of water consumption

w_t^d = desired level of water consumption

θ = coefficient of adjustment ($\theta < 0 < 1$)

t = current period

t-1 = previous period

Assuming that the relationship in equation (2) is a linear one, with coefficients a,b,c,d,e,f and g:

$$w_t^d = a + b*n_t + c*Y_t + d*p_t + e*q_t + f*PE_t + g*R_t \quad (2')$$

and substituting (2') in (1) the solution is:

$$w_t = \theta a + \theta b*n_t + \theta c*Y_t + \theta d*p_t + \theta e*q_t + \theta f*PE_t + \theta g*R_t + (1-\theta)*w_{t-1} \quad (3)$$

If the exogenous variables in equation (2') are replaced by the respective averages of the current and two previous periods, and if in addition equation (1) is modified as follows due to the seasonal variation in water consumption:

$$w_t = w_{t-3} + \theta(w_t^* - w_{t-3}) = (1-\theta)w_{t-3} + \theta * w_t^* \quad (1')$$

where t-3 is the 4-month period one year before, then the solution is:

$$w_t = \theta a + \theta b*(n_t+n_{t-1}+n_{t-2})/3 + \theta c*(Y_t+Y_{t-1}+Y_{t-2})/3 + \ldots + (1-\theta)*w_{t-3} \quad (4)$$

Equation (4) can be estimated as follows:

$$w_t = A + B*\bar{n}_t + C*\bar{Y}_t + D*\bar{p}_t + E*\bar{q}_t + F*\overline{PE}_t + G*\bar{R}_t + H*w_{t-3} \qquad (5)$$

where the $\bar{}$ symbol stands for the average of the current and two previous periods. The estimate of the coefficient of adjustment θ is given by

$$\hat{\theta} = 1 - \hat{H}$$

where $\hat{H}$ is the estimate of H.

2. Stock Adjustment Model

This model is described by equations (6) through (9):

$$w_t = F(n_t, Y_t, p_t, q_t, k_t, PE_t, R_t, S_t) \qquad (6)$$

$$S_t^d = G(n_t^e, Y_t^e, p_t^e, q_t^e, k_t^e, PE_t^e, R_t^e) \qquad (7)$$

$$n_t^e = n_{t-1}, \quad Y_t^e = Y_{t-1}, \quad p_t^e = p_{t-1}, \quad \text{etc.} \qquad (8)$$

$$S_t = S_{t-1} + \theta(S_t^d - S_{t-1}) \qquad (9)$$

Equation (6) states that water consumption depends on the current stock of water using appliances S_t and on its price k_t, in addition to current household size and income, prices of water and other goods, and weather variables. Equation (7) states that the desired current stock is a function of the expected future values of income, prices of water and other goods, price of the stock, and weather variables. Expectations are formed according to a certain mechanism, such as the one described in Equation (8), whereby the expected value of an exogenous variable is equal to the previous period's value (this process is called "static" expectations formation). Finally, the actual current stock is only partially adjusted toward the desired level, as described in Equation (9). The coefficient of adjustment, θ, is bounded between 0 and 1.

In order to illustrate the essence of the stock adjustment approach, the following linear model will be solved:

$$w_t = a + b*n_t + c*Y_t + d*S_t + e*p_t + f*q_t + g*k_t + h*PE_t + i*R_t \qquad (6')$$

$$S_t^d = j + m*n_t^e + o*Y_t^e + u*p_t^e + v*q_t^e + x*k_t^e + y*PE_t^e + z*R_t^e \qquad (7')$$

$$n_t^e = n_{t-1};\quad Y_t^e = Y_{t-1};\quad p_t^e = p_{t-1};\quad q_t^e = q_{t-1};\quad k_t^e = k_{t-1};$$
$$PE_t^e = PE_{t-1};\quad R_t^e = R_{t-1} \qquad (8')$$

$$s_t = s_{t-1} + \theta(s_t^d - s_{t-1}) = (1-\theta)*s_{t-1} + \theta*s_t^d \qquad (9')$$

Replacing (8′) in (7′):

$$S_t^d = j + m*n_{t-1} + o*Y_{t-1} + u*p_{t-1} + v*q_{t-1} + x*k_{t-1} + y*PE_{t-1} + z*R_{t-1} \qquad (10)$$

Replacing (10) in (9′) and (9′) in (6′):

$$w_t = a + d*\theta*j + b*n_t + d*\theta*m*n_{t-1} + c*Y_t + d*\theta*o*Y_{t-1} + e*p_t +$$
$$d*\theta*u*p_{t-1} + f*q_t + d*\theta*v*q_{t-1} + g*k_t + d*\theta*x*k_{t-1} + h*PE_t +$$
$$d*\theta*y*PE_{t-1} + i*R_t + d*\theta*z*R_{t-1} + d(1-\theta)s_{t-1} \qquad (11)$$

Since data on the stock of water-using appliances are usually not available, it is convenient to express s_{t-1} in (11) as a function of observable variables. This can be done using (6′) to compute the previous period's water consumption:

$$w_{t-1} = a + b*n_{t-1} + c*Y_{t-1} + d*S_{t-1} + e*p_{t-1} + f*q_{t-1} + g*k_{t-1} +$$
$$h*PE_{t-1} + i*R_{t-1} \qquad (12)$$

Rearranging (12):

$$s_{t-1} = 1/d*[w_{t-1} - a - b*n_{t-1} - c*Y_{t-1} - e*p_{t-1} - f*q_{t-1} -$$
$$g*k_{t-1} - h*PE_{t-1} - i*R_{t-1}] \qquad (13)$$

Replacing (13) in (11):

$$w_t = a + d*\theta*j - (1-\theta)*a + b*n_t + d*\theta*m*n_{t-1} - (1-\theta)*b*n_{t-1} + c*Y_t + d*\theta*o*Y_{t-1} - (1-\theta)*c*Y_{t-1} + e*p_t + d*\theta*u*p_{t-1} - (1-\theta)*e*p_{t-1} + f*q_t + d*\theta*v*q_{t-1} - (1-\theta)*f*q_{t-1} + g*k_t + d*\theta*x*k_{t-1} - (1-\theta)*g*k_{t-1} + h*PE_t + d*\theta*y*PE_{t-1} - (1-\theta)*h*PE_{t-1} + iR_t + d*\theta*z*R_{t-1} - (1-\theta)*i*R_{t-1} + (1-\theta)*w_{t-1} \quad (14)$$

Equation (14) can be estimated as follows:

$$w_t = A + B*n_t + C*n_{t-1} + D*Y_t + E*Y_{t-1} + F*p_t + G*p_{t-1} + H*q_t + I*q_{t-1} + J*k_t + K*k_{t-1} + L*PE_t + M*PE_{t-1} + N*R_t + O*R_{t-1} + u_t \quad (15)$$

where u_t is an error term.

APPENDIX E

Aggregation and the Representative Consumer Approach

1. Model of Appendix A

The solution to the model in Appendix A was:

$$w = A + B*(Y/p) + C*PE + D*R \quad (1)$$

Assume there are only two consumers whose demands are given by:

$$w^1 = A^1 + B^1*(Y^1/p) + C^1*PE + D^1*R \quad (2)$$

$$w^2 = A^2 + B^2*(Y^2/p) + C^2*PE + D^2*R \quad (3)$$

where the superscripts 1 and 2 identify each consumer. Equation (1) is the demand of the representative consumer whose income is

$$Y = (Y^1 + Y^2)/2 \quad (4)$$

PE, R and p are the same for all consumers. Replacing (4) in (1) and equating this to one half of the sum of (2) and (3) obtain:

$$w = A + B*(Y^1+Y^2)/2p + C*PE + D*R =$$

$$[A^1 + A^2 + (B^1*Y^1+B^2*Y^2)/p + (C^1+C^2)*PE + (D^1+D^2)*R]/2 \quad (5)$$

Equation (5) requires that the following conditions be met to validate the representative consumer approach:

$$A = A^1 + A^2 \quad (6)$$

$$B/2 = B^1 = B^2 \quad (7)$$

$$C = C^1 + C^2 \quad (8)$$

$$D = D^1 + D^2 \quad (9)$$

Conditions (6), (8) and (9) can always be met, but condition (7) is a stringent one, since it requires that equal increases in income result in equal water consumption increases across all consumers.

2. Model of Appendix B

The solution to the model in Appendix B was:

$$v = A + B*n + C*(Y/p) + D*PE + E*R + F*n*(q/p) \quad (10)$$

Using the same procedure as in 1. above, and defining:

$$n = (n^1+n^2)/2 \quad (11)$$

the following conditions are obtained:

$$A = A^1 + A^2 \quad (12)$$

$$B/2 = B^1 = B^2 \quad (13)$$

$$C/2 = C^1 = C^2 \quad (14)$$

$$D = D^1 + D^2 \quad (15)$$

$$E = E^1 + E^2 \quad (16)$$

$$F/2 = F^1 = F^2 \quad (17)$$

Conditions (12), (15) and (16) can always be met, whereas (13), (14) and (17) are very stringent. Equations (13) and (17) require that equal increases in household size result in equal water consumption increases across households. Equation (14) requires equal water consumption increases in response to equal income increases, across all households.

3. Model of Appendix C

The solution to the model in Appendix C was:

$$v = (p^r/(p^r+q^r))Y/p + (q^r/(p^r+q^r))(PE-R) \quad (18)$$

Using the same procedure as in 1. and 2. above, the following conditions are obtained:

$$p^{r'}/(p^{r'}+q^{r'}) = p^{r''}/(p^{r''}+q^{r''}) = p^r/(p^r+q^r) \quad (19)$$

$$(q^{r'}/(p^{r'}+q^{r'})) + (q^{r''}/(p^{r''}+q^{r''})) = 2q^r/(p^r+q^r) \quad (20)$$

where the consumers are nov identified by the superscripts ' and " to simplify the notation. Both conditions would be met if preferences were the same across consumers ($r' = r''$).

APPENDIX F

Supply and Demand Simultaneity

1. Simultaneity due to Cost and Price of Water

A supply/demand model is presented here to illustrate the potential for a simultaneity problem as a result of the pricing policy of the utility. The municipal water demand at time t is:

$$MW_t = W_t + NRW_t \qquad (1)$$

where:

MW = municipal water demand
W = residential water demand
NRW = non residential water demand

Assuming that the water demand of the non residential sector is a constant fraction (m) of the residential demand:

$$MW_t = W_t + m*W_t = (1+m)*W_t \qquad (2)$$

The residential water demand is equal to household demand w_t times the number of households N_t:

$$W_t = N_t*w_t \qquad (3)$$

Household demand is given by a linear model:

$$w_t = a_t + b_t*Y_t + c_t*SC_t + d_t*p_t + e_t*X_t \qquad (4)$$

where:

Y = household income
SC = service charge
p = water price
X = a weather variable

Household demand is assumed to be stable (a_t=a, b_t=b, c_t=c, d_t=d, e_t=e). Residential demand is:

$$W_t = N_t*(a + b*Y_t + c*SC_t + d*p_t + e*X_t) \qquad (5)$$

Municipal demand is:

$$MW_t = (1+m)*N_t*(a + b*Y_t + c*SC_t + d*p_t + e*X_t) \quad (6)$$

The utility uses (6) to forecast water demand in year T. The utility does so based on the following forecasts of the exogenous variables:

$$N_T^t = N_t + n*(T-t) \quad (7)$$

$$Y_T^t = Y_t + y*(T-t) \quad (8)$$

$$X_T^t = (X_L, X_M, X_H) \quad (9)$$

where N_T^t, Y_T^t and X_T^t stand for the forecasts of N_T, Y_T and X_T made at time t; n and y are historical rates of growth. The weather variable is assumed to exhibit no time trend, its average historical value is X_M and the extreme recorded values are X_L and X_H. The utility can set the values of the endogenous variables SC_T and p_T to control demand growth as shown below.

In this illustration the utility sets the system capacity based on expected demand and supply conditions under extreme climatic and hydrological conditions. Therefore, the utility uses X_H to forecast the system capacity QM_T at time T:

$$QM_T^t = (1+\rho)*MW_T^t(XH) =$$

$$= (1+\rho)*(1+m)*N_T^t*[a+b*Y_T^t+c*SC_T+d*p_T+e*X_H] \quad (11)$$

where (ρ) is a reserve capacity margin. The expected municipal consumption at time T under average weather conditions is:

$$Q_T^t = (1+m)*N_T^t*[a+b*Y_T^t+c*SC_T+d*p_T+e*X_M]$$

$$= QM_T + (1+m)*N_T^t*e*(X_M-X_H) \quad (12)$$

since $X_T^t=X_M$. If the forecasts of N and Y prove correct, the actual municipal consumption at time T will be lover than MW_T^t by the amount $(1+m)*N_T^t*e*(X_H-X_t)$, where X_t is the actual value of the weather variable at time T.

The supply source of the utility is subject to hydrological uncertainty which is also related to weather conditions. The utility must build

several sources of storage to prevent intertemporal and spatial imbalances between supply and demand. As a result of building reservoirs, treatment plants and distribution and collection networks, the utility has a long run cost of supply function:

$$C(QM_t,Q_t) = (r+\delta)*CC(QM_t) + z*Q_t \quad , \qquad Q_t < QM_t \tag{13}$$

where:

C = cost of supply
r = interest rate
δ = depreciation rate
CC = capacity cost
z = unit cost of operation to deliver Q_t to customers

The first term on the left hand side is the rental cost of capital (infrastructure) and the second term is the operation cost of delivery. In the short run, QM_t is fixed and the first term is thus a fixed cost $FC(QM_t)$ and the second term is the variable cost $VC(Q_t)$.

The capacity cost CC function is assumed to be:

$$CC(QM_t) = F + k*QB^3 + k*(QM_t-QB)^3 \tag{14}$$

Equation (14) states that the marginal cost of capacity is decreasing when QM_t is less than QB (economies of scale); it reaches a minimum when $QM_t=QB$ and it increases when QM_t grows beyond QB. In constrast, the marginal operation cost of delivery z^t in (13) is assumed to be constant.

Replacing (14) in (13):

$$\begin{aligned} C(QM_T^t,Q_T^t) &= (r+\delta)*[F + k*QB^3 + k*(QM_T^t-QB)^3] + z*Q_T^t \\ &= FC(QM_T^t) + VC(Q_T^t) \end{aligned} \tag{15}$$

The pricing policy of the utility determines whether a simultaneity problem exists. Three cases will be considered:

(i) Flat rate pricing

The utility distributes its expected total cost $C(QM_T^t,Q_T^t)$ equally among its customers in the form of a service charge. Assuming the non residential customers number $f*N_T^t$, the pricing rule is:

$$SC_T = C(QM_T^t, Q_T^t)/[N_T^t*(1+f)] \quad \text{and}$$

$$p_T = \text{zero} \tag{16}$$

$$SC_T = FC(QM_T^t)/[N_T^t*(1+f)] + VC(Q_T^t)/[N_T^t*(1+f)]$$
$$= (r+\delta)*[F+k*QB^3+k*(QM_T^t-QB)^3]/[N_T^t*(1+f)] +$$
$$z*(1+m)*N_T^t*w_T^t/[N_T^t*(1+f)] \tag{17}$$

Equation (17) has two terms on the right hand side. The second term is equal to $z*(1+m)*w_T^t/(1+f)$ and thus depends on w_T^t. The first term partially depends on w_T^t since QM_T^t can be expressed as:

$$QM_T^t = (1+m)*N_T^t*[a+b*Y_T^t+c*SC_T+d*p_T+e*X_H-e*X_M+e*X_M]$$
$$= (1+m)*N_T^t*[w_T^t+e*(X_H-X_M)] \tag{18}$$

Equation (17) can then be rewritten as:

$$SC_T = (r+\delta)*\{F+k*QB3+k*[(1+m)*N_T^t*(w_T^t+e*(X_H-X_M))-QB]^3\}/[N_T^t*(1+f)]$$
$$+ z*(1+m)*w_T^t/(1+f) = SC_T(w_T^t) \tag{19}$$

Equation (19) shows that the service charge is a function of the expected household demand w_T^t while Equation (4) expressed actual household demand as a function of the service charge. If the forecasts were always correct ($w_T^t=w_T$ for all T), then $SC_T=SC_T(w_T)$ and $w_T=w_T(SC_T)$. A simultaneity problem would exist under this pricing rule. The severity of the simultaneity problem would depend on the magnitude of the random deviations of the actual municipal demand from the expected one. The more accurate the forecasts, the more severe the simultaneity problem. However, the key forecast in this specific model is that of the weather variable and it is unlikely that that forecast will always be correct.

(ii) Average Cost Pricing

The pricing rule in this case is:

$$SC_T = \text{zero} \quad \text{and}$$

$$p_T = C(QM_T^t, Q_T^t)/Q_T^t \tag{20}$$

$$p_T = FC(QM_T^t)/Q_T^t + VC(Q_T^t)/Q_T^t =$$

$$(r+\delta)*\{F+k*QB^3+k*[(1+m)*N_T^t*(w_T^t+e*(X_H-X_M)-QB]^3\}/[(1+m)*N_T^t*w_T^t]$$

$$+ z*(1+m)*N_T^t*w_T^t/[(1+m)*N_T^t*w_T^t] \qquad (21)$$

The first term on the right hand side depends on w_T^t while the second one is equal to z. Therefore, the price p_T is a function of w_T^t, and since (4) expresses actual household demand w_T as a function of the price p_T, a simultaneity problem would exist. The same qualifications listed for the flat rate case also apply to this case.

(iii) Coase's pricing

An optimal pricing rule that maximizes consumers' welfare while allowing the utility to break even was derived by Coase and is described in McNeill (1989):

$$SC_T = FC(QM_T^t)/[(1+f)*N_T^t]$$

$$p_T = \text{marginal cost of supply} \qquad (22)$$

$$SC_T = (r+\delta)*\{F+k*QB^3+k*[(1+m)*N_T^t*(w_T^t+e*(X_H-X_M)-QB]^3\}/[(1+f)*N_T^t]$$

$$= SC_T(w_T^t) \qquad (23)$$

$$p_T = z \qquad (24)$$

A potential simultaneity problem is introduced through the service charge in this case.

In conclusion, as a result of the pricing policy followed by the utility, a simultaneity problem is bound to occur if the service charge and/or price are a function of the expected household demand, The severity of the simultaneity problem depends on the magnitude of the random deviations of the actual municipal demand from the expected demand. The more accurate the forecasts, the more severe the simultaneity problem. It also appears that the flat rate and average cost pricing policies used by most utilities tend to make the simultaneity problem more severe.

2. Simultaneity due to System Pressure

The water pressure H in the distribution network is related to municipal consumption MW as follows:

$$H = a + b*MW \qquad b < 0 \tag{25}$$

$$MW = (1+m)*W = (1+m)*N*w \tag{26}$$

Equation (25) states that pressure drops when municipal consumption increases; this relationship is particularly relevant in summer. Non-residential demand is assumed to be a constant proportion m of residential demand W. Household water demand w from each of the N households is in turn a function of pressure because of the influence of pressure on leakage and on the available supply at the household's water taps:

$$w = c + d*Z + e*H \tag{27}$$

where Z represents all the other explanatory variables (income, price, weather). From (25) and (26):

$$H = a + b*(1+m)*N*w = H(w) \tag{28}$$

and from (27):

$$w = w(H) \tag{29}$$

A simultaneity problem exists.

APPENDIX G

A Comprehensive Model of Residential Water Demand

1. Flow Adjustment Model

This model is a dynamic version of the Stone-Geary model of Appendix B. The utility function is rewritten as:

$$U(w_t,z_t) = K*n_t*[(w_t+R_t-PE_t)/n_t-w_{ot}]^a[z_t/n_t-z_{ot}]^b \quad (1)$$

where the subscript t refers to the current period. In order to simplify the solution of the model, it is assumed that the per capita subsistence levels of water and of the composite good do not change with time ($w_{ot}=w_o$, $z_{ot}=z_o$), even though in theory both levels should respond to changes in household income.

The budget constraint is:

$$Y_t = SC_t + p_t*w_t + q_t*z_t \quad (2)$$

The desired water consumption w_t^d can be obtained by maximizing the household utility function, given the levels of the explanatory variables, and subject to the budget constraint. The solution can be obtained using Lagrange's method:

$$w_t^d = b/(a+b)w_o n_t - a/(a+b)(q_t/p_t)z_o n_t + a/(a+b)(Y_t-SC_t)/p_t + b/(a+b)(PE_t-R_t) \quad (3)$$

The actual water consumption w_t adjusts partially towards the desired consumption level as follows:

$$w_t = w_{t-1} + \theta*(w_t^d-w_{t-1}) = (1-\theta)*w_{t-1} + \theta*w_t^d \quad (4)$$

Replacing (3) in (4):

$$w_t = \theta*b/(a+b)w_o n_t - \theta*a/(a+b)(q_t/p_t)z_o n_t + \theta*a/(a+b)(Y_t-SC_t)/p_t + \theta*b/(a+b)(PE_t-R_t) + (1-\theta)*w_{t-1} \quad (5)$$

The solution in (5) can be modified slightly to account for:

1. The use of trends in the determination of desired consumption.
2. Seasonal variation in water consumption.

The current levels of n, Y, SC, p and q in (5) can be replaced by the corresponding averages of the current and two previous periods to take time trends into account. The current levels and not the trends should be used in the case of the weather variables PE and R, given the seasonality of water consumption. Seasonal variation also suggests that the actual consumption in the previous period should be replaced in (4) by the actual consumption in the corresponding season of the previous year, i.e. three periods before the current one. As a result of these modifications the solution is:

$$w_t = \Theta w_o a/(a+b)\bar{n}_t + \Theta z_o a/(a+b)(\bar{q}_t/\bar{p}_t)\bar{n}_t + \Theta a/a+b(\bar{Y}_t-\overline{SC}_t)/\bar{p}_t + \Theta b/(a+b)(PE_t-R_t) + (1-\Theta)w_{t-3} \quad (6)$$

where the $\bar{}$ symbol stands for the average of the current and two previous periods. Equation (6) can be estimated as follows:

$$w_t = A + B*\bar{n}_t + C*(\bar{q}_t/\bar{p}_t)\bar{n}_t + D*\bar{Y}_t/\bar{p}_t + E*\overline{SC}_t/\bar{p}_t + F*PE_t + G*R_t + H*w_{t-3} \quad (7)$$

The coefficient of adjustment Θ and the subsistence level w_o can be estimated from:

$$\hat{\Theta} = 1 - \hat{H}, \qquad \hat{w}_o = \hat{B}/\hat{D},$$

where $\hat{B}$, $\hat{D}$ and $\hat{H}$ are the estimates of the coefficients of equation (7) obtained by the Least Squares or Maximum Likelihood methods.

2. Stock Adjustment Model

The utility function of the flow adjustment model is rewritten as:

$$U(w_t,z_t,s_t) = K*n_t*s_t*[(w_t+R_t-PE_t)/(n_t*s_t)-w_{ot}/s_t]^a[z_t/n_t-z_{ot}]^b$$
$$= K*n_t*s_t^{1-a}[(w_t+R_t-PE_t)/n_t-w_{ot}]^a[z_t/n_t-z_{ot}]^b \quad (8)$$

where s_t is the current stock of water using appliances, which is fixed in the short run but free to vary in the long run.

Equation (8) states that each household member allocates his/her share of water consumption evenly among the household's stock of water-using appliances. As the stock grows, less water will be allocated to each appliance from a fixed total amount of water; utility will increase however, given the increased stock and the diminishing marginal utility of

the water allocated to each appliance.

In order to facilitate the solution of the model, the demand for the composite good z_t is not assumed to depend on an associated stock. As in the flow adjustment model, it is assumed that the subsistence levels w_{ot} and z_{ot} do not change with time ($w_{ot}=w_o$, $z_{ot}=z_o$). In theory, w_{ot} should change in response to changes in the per capita stock of water using appliances, but the solution to the model is non-linear under that assumption.

The budget constraint is:

$$Y_t = SCt + p_t*w_t + q_t*z_t + k_t*s_t \tag{9}$$

where k_t is the current rental price of the stock of water using appliances.

The first stage of the consumer's problem is to select the optimal levels of water consumption, composite good consumption, and stock of water-using appliances based on the expected values of the explanatory variables. The levels that maximize the long run utility function, subject to the budget constraint, obtained using Lagrange's method are:

$$w_t^d = a/(1+b)(Y_t^e-SC_t^e)/p_t^e + (1+b-a)/(1+b)(PE^e-R^e) + (1+b-a)/(1+b)n_t^e w_o - a/(1+b)(q_t^e/p_t^e)n_t^e z_o \tag{10}$$

$$z_t^d = b/(1+b)(Y_t^e-SC_t^e)/q_t^e - b/(1+b)(p_t^e/q_t^e)(PE_t^e-R_t^e) - b/(1+b)(p_t^e/q_t^e)n_t^e w_o + 1/(1+b)n_t^e z_o \tag{11}$$

$$s_t^d = (1-a)/(1+b)(Y_t^e-SC_t^e)/k_t^e - (1-a)/(1+b)(p_t^e/k_t^e)(PE_t^e-R_t^e) - (1-a)/(1+b)(p_t^e/k_t^e)n_t^e w_o - (1-a)/(1+b)(q_t^e/k_t^e)n_t^e z_o \tag{12}$$

where the superscripts d and e stand for desired and expected value respectively. A static mechanism of expectations formation has been assumed whereby the expected value of each explanatory variable is equal to the previous period's value:

$$n_t^e=n_{t-1},\quad Y_t^e=Y_{t-1},\quad SC_t^e=SC_{t-1},\quad p_t^e=p_{t-1},\quad q_t^e=q_{t-1},$$
$$k_t^e=k_{t-1},\quad PE_t^e=PE_{t-1},\quad R_t^e=R_{t-1} \tag{13}$$

An adaptive expectations approach could have been used as an alternative. Expected values in this case are a weighted average of the values in more than one previous periods. Static expectations were used here to simplify the model solution. Both static and adaptive mechanisms allow for regular updates of the forecasts based on the latest information.

Replacing (13) in (12) obtain:

$$s_t^d = (1-a)/(1+b)(Y_{t-1}-SC_{t-1})/k_{t-1} - (1-a)/(1+b)(p_{t-1}/k_{t-1})(PE_{t-1}-R_{t-1}) - (1-a)/(1+b)(p_{t-1}/k_{t-1})n_{t-1}v_o - (1-a)/(1+b)(q_{t-1}/k_{t-1})n_{t-1}z_o \quad (14)$$

Hovever, the actual current stock s_t is assumed to be only partially adjusted towards the optimal level:

$$s_t = s_{t-1} + \theta(s_t^d - s_{t-1}) = (1-\theta)s_{t-1} + \theta s_t^d \qquad 0 < \theta < 1 \quad (15)$$

where s_{t-1} is the previous period stock and θ is the coefficient of adjustment.

The actual value of water consumption in a given period can now be determined. Since s_t is fixed in the short run, the consumer's problem is:

Select v_t and z_t to maximize $U(s_t=s_t, v_t, z_t)$

given $K, a, b, v_o, z_o, s_t=s_t, n_t, Y_t, p_t, q_t, k_t, PE_t, R_t$

and subject to $Y_t = SC_t + p_t*v_t + q_t*z_t + k_t*s_t$

The solution is:

$$v_t^d = a/(a+b)(Y_t-SC_t)/p_t - a/(a+b)(k_t/p_t)s_t + b/(a+b)(PE_t-R_t) + b/(a+b)n_t v_o - a/(a+b)(q_t/p_t)n_t z_o \quad (16)$$

Before replacing (15) in (16), s_{t-1} can be expressed as a function of the exogenous variables, using (16) to compute the previous period's water consumption:

$$v_{t-1} = a/(a+b)(Y_{t-1}-SC_{t-1})/p_{t-1} - a/(a+b)(k_{t-1}/p_{t-1})s_{t-1} + b/(a+b)(PE_{t-1}-R_{t-1}) + b/(a+b)n_{t-1}v_o - a/(a+b)(q_{t-1}/p_{t-1})n_{t-1}z_o \quad (17)$$

Rearranging (17):

$$s_{t-1} = (Y_{t-1}-SC_{t-1})/k_{t-1} + b/a(p_{t-1}/k_{t-1})(PE_{t-1}-R_{t-1}) + b/a(p_{t-1}/k_{t-1})n_{t-1}w_o - (q_{t-1}/k_{t-1})n_{t-1}z_o - (a+b)/a(p_{t-1}/k_{t-1})w_{t-1} \quad (18)$$

Replacing (18) in (15), and (15) in (16):

$$w_t = A + B^*(Y_t-SC_t)/p_t + C(k_t/p_t)(Y_{t-1}-SC_{t-1})/k_{t-1} + D(PE_t-R_t) + E(k_t/k_{t-1})/(p_t/p_{t-1})(PE_{t-1}-R_{t-1}) + Fn_t + G(k_t/k_{t-1})/(p_t/p_{t-1})n_{t-1} + H(q_t/p_t)n_t + I(k_t/k_{t-1})/(p_t/q_{t-1})n_{t-1} + J(k_t/k_{t-1})/(p_t/p_{t-1})w_{t-1} \quad (19)$$

where:

A = zero B = a/(a+b) C = -a/(a+b)[(1-θ)+θ(1-a)/(1+b)]

D = b/(a+b) E = -a/(a+b)[(1-θ)b/a+θ(1-a)/(1+b)] F = w_o(b/(a+b))

G = -a/(a+b)w_o[(1-θ)b/a+θ(1-a)/(1+b)] H = -z_o(a/(a+b))

I = a/(a+b)z_o[(1-θ)+θ(1-a)/(1+b)] J = 1-θ

Equation (19) can be estimated by the Least Squares or Maximum Likelihood methods. The coefficient of adjustment and subsistence levels can be computed from:

$$\hat{\theta} = 1-\hat{J} \qquad \hat{w}_o = \hat{F}/\hat{B} = \hat{G}/\hat{E} \qquad \hat{z}_o = \hat{H}/\hat{B} = \hat{I}/\hat{C} \quad (20)$$

where $\hat{B}$, $\hat{C}$, $\hat{D}$, $\hat{E}$, $\hat{F}$, $\hat{G}$, $\hat{H}$, $\hat{I}$ and $\hat{J}$ are the estimates of the respective coefficients, obtained by the Least Squares or Maximum Likelihood methods.

If the following restrictive assumption is made:

$$p_t/p_{t-1} = q_t/q_{t-1} = k_t/k_{t-1} \quad (21)$$

that is, if the inflation rate is the same for all prices, Equation (19) simplifies to:

$$w_t = A + B(Y_t-SC_t)/p_t + C(Y_{t-1}-SC_{t-1})/p_{t-1} + D(PE_t-R_t) + E(PE_{t-1}-R_{t-1}) + Fn_t + Gn_{t-1} + H(q_t/p_t)n_t + I(q_{t-1}/p_{t-1})n_{t-1} + Jw_{t-1} \quad (22)$$

The price of the stock, k_t, does not appear in (22). The estimation of (22) is simpler, but the model is a restricted version of the more general model of Equation (19).

APPENDIX H

Data Appendix

Table H.1	Greater Victoria Potential Evapotranspiration
Table H.2	Oak Bay Distribution Network Pressure
Table H.3	Wages Index
Table H.4	Fuel Price Index
Table H.5	Materials Price Index
Table H.6	5-year Mortgage Rate
Table H.7	Residential Capital Stock. Price Index

TABLE H.1 Greater Victoria Potential Evapotranspiration.
Victoria Gonzalez Heights Station, 1961-1986.

inches

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1961	0.86	0.95	1.42	1.89	2.95	3.83	4.22	3.94	2.71	1.74	0.84	0.59
1962	0.65	0.81	1.06	2.00	2.63	3.37	3.75	3.46	2.84	1.94	1.17	0.85
1963	0.41	1.15	1.30	1.92	3.13	3.47	3.87	3.57	3.06	2.04	1.04	0.78
1964	0.76	0.82	1.24	1.64	2.61	3.31	3.79	3.59	2.60	1.98	0.83	0.45
1965	0.58	0.75	1.35	2.00	2.52	3.41	4.04	3.60	2.69	2.04	1.16	0.64
1966	0.66	0.86	1.31	2.00	2.77	3.35	3.63	3.64	2.88	1.78	1.10	0.87
1967	0.74	0.90	1.11	1.67	2.81	3.66	4.07	4.04	3.13	1.87	1.09	0.65
1968	0.63	1.08	1.56	1.76	3.05	3.40	4.28	3.46	2.73	1.70	1.06	0.48
1969	0.23	0.69	1.44	1.85	3.36	3.96	3.95	3.46	2.72	1.87	1.04	0.81
1970	0.65	1.05	1.44	1.70	2.75	3.74	3.92	3.53	2.60	1.73	1.00	0.54
1971	0.59	0.74	0.96	1.85	2.86	3.11	3.93	3.86	2.72	1.72	0.98	0.40
1972	0.37	0.72	1.32	1.51	3.18	3.36	4.15	3.80	2.51	1.75	1.10	0.50
1973	0.58	0.85	1.32	1.89	2.96	3.40	3.84	3.34	2.90	1.73	0.75	0.81
1974	0.57	0.81	1.26	1.93	2.57	3.39	3.82	3.66	3.35	1.96	1.06	0.87
1975	0.55	0.61	1.14	1.65	2.80	3.26	3.86	3.34	3.09	1.75	0.98	0.67
1976	0.73	0.73	0.97	1.99	2.68	3.09	3.83	3.26	2.89	1.88	1.12	0.83
1977	0.57	1.09	1.22	2.09	2.56	3.48	3.73	3.82	2.63	1.76	0.87	0.58
1978	0.69	0.90	1.52	2.00	2.85	3.92	4.00	3.69	2.60	2.01	0.72	0.52
1979	0.33	0.64	1.56	2.01	3.07	3.44	4.06	3.59	3.12	2.03	1.05	0.90
1980	0.41	0.91	1.29	2.16	2.88	3.27	3.85	3.45	2.70	2.03	1.13	0.84
1981	1.02	0.91	1.73	1.95	2.93	3.27	3.89	3.96	2.85	1.76	1.18	0.68
1982	0.59	0.79	1.24	1.77	2.79	3.75	3.78	3.69	2.99	1.93	0.84	0.70
1983	0.96	1.09	1.72	2.15	3.34	3.61	4.02	3.84	2.71	1.76	1.20	0.41
1984	0.81	1.02	1.66	1.96	2.65	3.27	4.06	3.65	2.78	1.66	0.97	0.42
1985	0.54	0.69	1.20	1.85	2.91	3.54	4.21	3.79	2.73	1.85	0.50	0.52
1986	0.96	0.77	1.61	1.81	2.88	3.68	3.67	4.08	2.80	2.00	1.02	0.78

Source: Atmospheric Environment Service, Environment Canada (May 1988).

TABLE H.2 Oak Bay Distribution Network Pressure

pounds per square inch

	MINIMUM			MAXIMUM		
YEAR	JAN-APR	MAY-AUG	SEP-DEC	JAN-APR	MAY-AUG	SEP-DEC
1976	74	62	78	98	97	98
1977	76	66	78	98	107	96
1978	75	50	79	102	118	95
1979	77	48	79	94	92	98
1980	76	61	76	108	95	105
1981	78	70	84	101	98	99
1982	72	75	76	86	92	106
1983	82	78	81	93	93	94
1984	82	51	80	95	96	96
1985	78	64	78	98	93	96
1986	63	52	63	100	90	102

Readings recorded at Municipal Hall's gage.

TABLE H.3 Wages Index (1981=100.0)

YEAR	INDEX
1977	66.1
1978	70.3
1979	77.6
1980	87.3
1981	100.0
1982	109.8
1983	119.5
1984	131.0
1985	136.6
1986	147.9

Source: Statistics Canada Catalogue 56-203 "Telephone Statistics".

TABLE H.4 Fuel Price Index (1981=100.0)

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1976	35.7	35.6	35.6	35.7	36.2	36.3	36.4	36.4	39.9	40.1	40.1	40.2
1977	40.3	40.4	42.0	43.1	43.0	43.4	43.4	43.3	45.7	45.9	45.6	45.2
1978	45.0	45.0	47.4	47.9	48.1	47.7	47.7	49.4	52.7	52.6	52.5	52.9
1979	53.1	53.7	53.6	53.8	53.8	54.2	55.4	56.6	59.9	60.5	61.7	63.3
1980	63.7	64.4	67.0	67.8	68.0	68.0	68.9	70.4	70.3	76.2	79.5	79.6
1981	86.9	87.9	93.2	94.4	96.9	101.8	103.2	103.3	107.0	108.4	108.0	109.0
1982	108.4	109.6	115.2	116.6	117.6	117.6	116.6	118.1	124.6	124.6	124.1	122.8
1983	116.1	113.3	124.6	123.3	121.8	126.9	127.4	128.4	128.4	127.1	125.5	127.6
1984	131.0	130.1	129.0	128.6	127.6	127.6	130.1	131.2	130.8	130.1	134.5	137.5
1985	139.9	139.8	139.4	138.6	138.2	137.4	137.0	135.7	136.4	137.1	138.3	140.6
1986	140.3	138.9	128.9	113.7	104.4	102.3	100.5	97.8	96.6	96.0	96.8	96.0

Source: Statistics Canada Catalogue 62-011 "Industry Price Indexes".

TABLE H.5 Materials Price Index (1981=100.0)

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1977	61.6	62.6	64.3	65.7	66.9	66.0	65.7	64.5	64.5	64.7	65.9	66.2
1978	67.4	68.3	69.1	70.9	73.1	74.6	73.4	74.1	74.8	77.2	77.2	78.1
1979	80.5	85.4	88.3	87.2	87.8	89.8	89.7	88.4	90.8	92.3	92.3	93.8
1980	100.4	100.8	96.2	93.6	95.7	95.3	97.3	99.8	100.2	104.0	103.8	102.2
1981	102.8	101.2	100.6	100.4	100.1	101.4	101.5	101.8	100.3	98.5	96.6	94.8
1982	95.6	97.6	96.8	98.1	100.1	99.3	98.5	97.2	97.6	95.3	94.0	94.4
1983	95.2	97.2	96.8	99.0	99.4	99.0	98.2	99.8	99.2	99.3	99.0	99.9
1984	101.5	102.5	104.9	105.7	106.1	105.5	105.0	102.8	101.5	100.1	100.8	100.6
1985	100.4	101.3	101.8	101.7	101.3	100.8	99.2	98.6	98.2	99.4	100.4	101.6
1986	103.0	102.6	103.3	102.3	102.7	103.3	104.2	104.9	105.5	104.7	105.8	105.0

Source: Statistics Canada Catalogue 62-011 "Industry Price Indexes".

TABLE H.6 5-year Mortgage Rate, 1973-1986.

(percentages)

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1973	9.00	9.00	9.00	9.25	9.25	9.50	9.75	9.75	10.00	10.00	10.00	10.25
1974	10.00	10.00	10.00	10.50	11.50	11.50	11.50	11.75	12.00	12.00	12.00	12.00
1975	11.50	10.75	10.50	10.75	11.00	11.00	11.25	11.50	11.75	12.00	12.00	12.00
1976	12.00	11.75	11.75	12.00	12.00	12.00	12.00	12.00	11.75	11.75	11.25	10.75
1977	10.75	10.25	10.25	10.25	10.25	10.25	10.25	10.25	10.25	10.25	10.25	10.25
1978	10.25	10.25	10.25	10.25	10.25	10.25	10.25	10.25	10.75	11.25	11.25	11.25
1979	11.25	11.25	11.00	11.00	11.25	11.25	11.75	12.00	12.75	14.75	14.75	13.25
1980	13.25	14.50	15.75	16.75	13.00	13.00	13.50	14.00	14.50	15.00	15.00	16.00
1981	15.50	15.50	16.00	17.00	18.50	18.75	20.00	21.75	21.75	20.25	17.75	17.75
1982	18.75	19.50	19.50	19.50	19.50	19.75	19.25	18.25	17.25	15.75	14.75	14.75
1983	13.50	13.50	13.50	13.25	13.00	13.00	13.50	14.00	13.50	13.00	12.50	12.50
1984	12.50	12.50	13.25	13.50	14.50	14.50	15.25	14.25	13.75	13.50	13.00	12.50
1985	12.25	13.25	13.25	12.50	12.25	11.75	11.75	11.75	11.75	11.75	11.75	11.50
1986	11.75	12.00	11.50	11.00	10.50	11.00	11.00	11.00	11.25	11.25	11.25	11.00

Source: Statistics Canada, Bank of Canada monthly reports.

TABLE H.7 Residential Capital Stock. Price Index (1981=100.0)

YEAR	INDEX
1961	25.6
1962	25.5
1963	26.1
1964	27.2
1965	28.7
1966	30.6
1967	32.4
1968	32.7
1969	34.3
1970	35.0
1971	37.2
1972	39.9
1973	46.1
1974	55.1
1975	62.5
1976	69.0
1977	76.9
1978	83.0
1979	88.9
1980	92.1
1981	100.0
1982	94.1
1983	98.0
1984	101.5
1985	104.3
1986	108.0

Source: Statistics Canada Catalogues 13-211 and 13-568 "Fixed Capital Stocks and Flows" (Residential).

APPENDIX I

TWO-STAGE LEAST SQUARES. REGRESSION STATISTICS

Indoor-plus-outdoor Water Consumption Per House.

Indoor Water Consumption Per House.

Outdoor Water Consumption Per House.

TABLE I.1 Regression Statistics (Two-Stage Least Squares).
Indoor-plus-Outdoor Water Consumption per House.

MODEL	1.S	1.D	2.S	2.D	3.S	3.D
	LINEAR		LOG-LOG		COMPREHENSIVE	
.	static	dynamic	static	dynamic	static	dynamic .
DEPENDENT VARIABLE	PHT	PHT	PHT	PHT	PHT	PHT .
INDEPENDENT VARIABLES						
NPEOP	**684.34**	444.88	**2.0423**	0.85987	752.36	456.75
	(2.65)	(1.58)	**(2.37)**	(1.09)	(1.78)	(1.11)
RDINC	**0.071545**	0.045363	0.28307	0.20850	-	-
	(2.10)	(1.37)	(1.01)	(1.01)		
RSCP	28.220	17.500	0.17079	0.073390	-	-
	(1.06)	(0.70)	(0.35)	(0.20)		
RMPP	-131650.	-96384.	**-.78948**	-.44245	-	-
	(1.25)	(1.04)	**(2.24)**	(1.66)		
GIPRP	-	-	-	-	.00021895	.00014332
					(1.90)	(1.26)
SCPRP	-	-	-	-	.054755	.028852
					(0.81)	(0.47)
NPMRP	-	-	-	-	-.45558	-.11323
					(0.46)	(0.13)
PE	**284.96**	**120.10**	**0.57041**	0.16887	**280.94**	**119.49**
	(7.95)	**(3.14)**	**(5.53)**	(1.87)	**(7.85)**	**(3.16)**
RAIN	**-271.02**	**-122.68**	**-.34396**	**-.15593**	**-267.14**	**-123.20**
	(4.45)	**(2.13)**	**(3.46)**	**(2.04)**	**(4.38)**	**(2.15)**
LAGPHT	-	-.090270	-	-.025402	-	-.090290
		(1.00)		(0.32)		(1.00)
LAGPHT3	-	**0.54316**	-	**0.69040**	-	**0.54568**
		(5.23)		**(7.42)**		**(5.29)**
CONSTANT	-1593.3	-969.16	-3.0007	-3.2221	-1705.4	-1139.8
	(1.34)	(0.85)	(0.71)	(1.01)	(1.69)	(1.11)
BUSE R2	0.5926	0.7544	0.5410	0.8025	0.5895	0.7553
SEE	0.88999	0.86483	0.88781	0.87844	0.88829	0.86402
T-df	83	81	83	81	83	81
F-ndf	6	8	6	8	6	8
F-ddf	83	81	83	81	83	81
F-statistic	20.1	31.1	16.3	41.1	19.9	31.3

Continued.

TABLE I.1 Continued.
Indoor Water Consumption per House.

MODEL	1.S	1.D	2.S	2.D	3.S	3.D
	LINEAR		LOG-LOG		COMPREHENSIVE	
.	static	dynamic	static	dynamic	static	dynamic .
DEPENDENT VARIABLE	PHI	PHI	PHI	PHI	PHI	PHI .
INDEPENDENT VARIABLES						
NPEOP	760.22	532.91	2.5385	1.9151	857.79	588.53
	(6.22)	(4.40)	(4.12)	(3.36)	(6.14)	(4.05)
RDINC	.012277	.0093347	.0097992	.068072	-	-
	(1.21)	(1.20)	(0.06)	(0.58)		
RSCP	30.647	20.665	0.37708	0.35870	-	-
	(3.58)	(2.82)	(1.51)	(1.65)		
RMPP	-96413.	-83569.	-.56190	-.43838	-	-
	(2.71)	(3.54)	(2.37)	(3.04)		
GIPRP	-	-	-	-	.73982E-4	.54638E-4
					(2.41)	(2.09)
SCPRP	-	-	-	-	.083904	.055487
					(4.19)	(3.06)
NPMRP	-	-	-	-	-.19983	-.034033
					(0.73)	(0.17)
PE	-	-	-	-	-	-
RAIN	-	-	-	-	-	-
LAGPHI	-	0.38805	-	0.48968	-	0.33625
		(3.66)		(4.93)		(3.07)
DIO	25.658	32.757	.041556	.054904	22.123	31.780
	(2.11)	(2.54)	(2.04)	(2.52)	(1.88)	(2.55)
CONSTANT	-1350.5	-960.36	.0088513	-2.4100	-1831.2	-1362.4
	(3.28)	(2.82)	(0.003)	(1.29)	(5.05)	(3.97)
BUSE R2	0.4037	0.6996	0.3064	0.7111	0.4743	0.7011
SEE	0.90936	0.89982	0.92002	0.90571	0.91196	0.90123
T-df	84	83	84	83	84	83
F-ndf	5	6	5	6	5	6
F-ddf	84	83	84	83	84	83
F-statistic	11.4	32.2	7.4	34.0	15.2	32.4

Continued.

TABLE I.1 Continued.
Outdoor Water Consumption per House.

MODEL	1.S	1.D	2.S	2.D	3.S	3.D
	LINEAR		LOG-LOG		COMPREHENSIVE	
.	static	dynamic	static	dynamic	static	dynamic .
DEPENDENT VARIABLE	PHO	PHO	PHO	PHO	PHO	PHO .
INDEPENDENT VARIABLES						
NPEOP	-556.10	-317.86	-.16856	4.7995	-542.84	-296.34
	(0.97)	(0.52)	(0.05)	(1.02)	(0.73)	(0.35)
RDINC	.032945	.027914	1.6734	**2.6473**	-	-
	(0.70)	(0.51)	(1.57)	**(2.06)**		
RSCP	-29.635	-9.6425	.079190	2.0600	-	-
	(0.68)	(0.19)	(0.04)	(0.82)		
RMPP	152500.	171750.	1.4495	0.87854	-	-
	(0.65)	(0.68)	(0.57)	(0.34)		
GIPRP	-	-	-	-	.39257E-4	.23543E-4
					(0.24)	(0.12)
SCPRP	-	-	-	-	-.094782	-.057730
					(0.84)	(0.45)
NPHRP	-	-	-	-	-.35314	-.43100
					(0.21)	(0.23)
PE	**455.36**	253.11	**6.2274**	**5.0016**	**448.41**	253.54
	(3.59)	(1.54)	**(5.49)**	**(3.38)**	**(3.67)**	(1.56)
RAIN	-120.81	-184.38	0.26820	0.13677	-134.61	-206.96
	(0.72)	(1.04)	(0.48)	(0.23)	(0.82)	(1.19)
LAGPH03	-	0.29624	-	0.14358	-	0.28629
		(1.61)		(1.45)		(1.56)
CONSTANT	283.49	-32.872	-8.3394	-28.872	1252.4	1134.2
	(0.13)	(0.01)	(0.40)	(1.28)	(0.61)	(0.51)
BUSE R2	0.7164	0.7577	0.7535	0.8212	0.7263	0.7617
SEE	0.78353	0.74092	0.77932	0.74217	0.78591	0.74312
T-df	38	37	38	37	38	37
F-ndf	6	7	6	7	6	7
F-ddf	38	37	38	37	38	37
F-statistic	16.0	16.5	19.4	24.3	16.8	16.9

Continued.

TABLE I.1 Concluded.

NOTES: All variables and definitions as per Table 3 in the main text, except for the economic variables RSC, RMP, GIPR, SCPR and NPMR which have been replaced by RSCP, RMPP, GIPRP, SCPRP and NPMRP respectively. The new variables have been obtained at the first stage of the Two-stage estimation procedure. Each of the five variables has been regressed on the number of dwellings served, interest rate, a price index of wages, fuel and materials, time, municipal dummy variables and on the other exogenous variables of the demand equation (NPEOP, RDINC, PE and RAIN). The estimated variables (RSCP, RMPP, GIPRP, SCPRP and NPMRP) have been substituted in the demand equation to obtain the estimates reported in the preceding tables.

APPENDIX J

Table J.1 A Comparison of the Stock-Adjustment and Flow-Adjustment Models. Regression Statistics.

MODEL	<-------STOCK ADJUSTMENT---->			<-------FLOW ADJUSTMENT------->		
DEPENDENT VARIABLE	current period PHT			current period PHT		
INDEPENDENT VARIABLES	current period	previous period	previous 3 periods	current & previous 2	previous period	3 periods before
NPEOP	2274.0 (1.33)	1323.5 (1.66)	-3237.8 (1.88)	237.32 (1.78)	-	-
GIPR	.00014640 (0.18)	.00088754 (0.57)	-.0010071 (1.04)	.00008062 (1.56)	-	-
SCPR	.017833 (0.28)	.027854 (0.31)	-.058772 (0.70)	.011130 (0.50)	-	-
NPMR	1.9176 (0.29)	-9.4405 (0.79)	7.6171 (1.16)	-.16988 (0.38)	-	-
PE	**277.08** **(3.29)**	51.466 (0.56)	1977.2 (1.47)	**104.89** **(2.82)**	-	-
RAIN	-191.11 (1.42)	-62.046 (0.44)	-332.33 (0.08)	**-117.96** **(2.09)**	-	-
LAGPHT	-	-0.28616	-	-	-.067848 (0.75)	**0.58795** **(5.70)**
CONSTANT	-4641.8 (1.19)	-	-	-348.36 (0.98)	-	-
BUSE R2	0.7110			0.7491		
SEE	0.63096			0.86229		
T-df	70			81		
F-ndf	19			8		
F-ddf	70			81		
F-statistic	9.06			30.2		

Continued.

Table J.1 Concluded.

NOTES: Each independent variable of the stock-adjustment model is pre-multiplied by a ratio of the prices of water, other goods, and stock of water-using appliances, as described in Appendix G.

The PE and RAIN variables in the flow-adjustment models correspond to the current period only.